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Laser arrays of partially coherent beams with multi-Gaussian correlation function

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ABSTRACT

We study the laser arrays which are constructed by partially coherent beams with multi-Gaussian correlation function, i.e., circular, rectangular and elliptical multi-Gaussian Schell-model (MGSM) beam arrays. Analytical propagation formulas for circular, rectangular and elliptical MGSM beam arrays through a stigmatic ABCD optical system and in turbulent atmosphere are derived, with which the propagation properties of such laser arrays may be easily determined. It is found that in the near field, the irradiance distributions of circular, rectangular and elliptical MGSM beam arrays are mainly controlled by the distribution of beam arrays in the source plane. In the far field, the irradiance distributions of circular, rectangular and elliptical MGSM beam arrays in free space are determined by their correlation functions respectively; while in turbulent atmosphere, the correlation functions and turbulent atmosphere all influence the irradiance distributions, especially after enough long distance propagation, the turbulent atmosphere may change the irradiance distributions from non-circular symmetry to circular symmetry.

1. Introduction

In the past decades, partially coherent beams have received widespread attention due to their important applications in free-space optical communications, remote detection, optical imaging, particle scattering and particle trapping, etc. [1–11]. Most of the researches are focused on partially coherent beams with conventional Gaussian correlation function (e.g., Gaussian Schell-model beams). The intensity distribution of a partially coherent beam with Gaussian correlation function in the far field remains Gaussian beam profile, which is determined by the well-known reciprocity relations [1]. Since Gori et al. discussed the condition for devising a genuine correlation function of a scalar or electromagnetic partially coherent beam [12,13], more and more attention has been paid to partially coherent beams with non-conventional correlation functions (e.g., nonuniformly correlated Gaussian Schell-model beams, MGSM beams, cosine-Gaussian Schell-model beams and Laguerre-Gaussian Schell-model beams, etc.) [14–36]. Circular, rectangular and elliptical MGSM beams were introduced successively, whose intensity distributions in the far field will preserve circular, rectangular and elliptical flat-topped beam profiles, respectively [22–24]. Though there are flat-topped beams who have circular, rectangular and elliptical flat-topped intensity profiles close to the source, such profiles will gradually become Gaussian after propagation [37–39]. MGSM beam carrying a vortex phase which is called MGSM vortex beam was proposed in [25]. A generalized MGSM beam who is capable of producing dark hollow or flat-topped beam profile in the far field was studied in [26]. The study of MGSM beams has also been extended to electromagnetic and non-stationary fields [27,28]. Propagation properties of MGSM beams have been explored extensively in [29–36].

On the other hand, laser arrays have been studied extensively due to the fact that low power output of a single laser beam hampers its applications [40,41]. It was also found that laser arrays can reduce scintillation [42,43], which will be very useful in free-space optical communications. A laser array can be constructed through coherent or incoherent superposition of a series of off-axis laser beams. A variety of linear, rectangular and radial laser arrays have been developed, and the propagation properties of such laser arrays have attracted much attention [44–50]. The study of superposition of off-axial laser beams has been extended from coherent case to partially coherent case. Partially coherent beam arrays which are constructed by scalar or electromagnetic partially coherent beams with conventional Gaussian correlation function have

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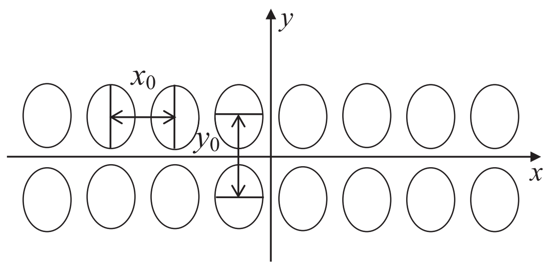

Fig. 1. Schematic illustration of a rectangular partially coherent beam array.

been studied extensively [51–57]. In this paper we extend the superposition of partially coherent beams with conventional Gaussian correlation function to that with non-conventional correlation functions. As a typical example, the incoherent superposition of partially coherent beams with multi-Gaussian correlation function is investigated. Analytical expressions for the propagation of circular, rectangular and elliptical MGSM beam arrays through a stigmatic ABCD optical system and in turbulent atmosphere are derived, with which their propagation properties in free space and turbulent atmosphere are studied comparatively. Some interesting and useful results are found.

2. Propagation of circular, rectangular and elliptical MGSM beam arrays through a stigmatic ABCD optical system

Assume that a rectangular partially coherent beam array can be synthesized through incoherent superposition of $E\times F$ equal off-axis partially coherent beams, which are positioned at $z=0$ as shown in Fig. 1. The separation distances are x_0 and y_0. In this paper, we assume E and F to be even numbers, but the extension to odd numbers is straightforward. $E=8$, $F=2$, $x_0=50$ and $y_0=50$ are kept fixed throughout this paper.

Within the validity of paraxial approximation, propagation of the cross-spectral density function of an off-axis partially coherent beam with e and f subscripts through a stigmatic $ABCD$ optical system can be studied with the help of the following generalized Collins integral [1–3]

$$
\begin{aligned}
&W_{ef}\left(\rho_{1x},\rho_{1y},\rho_{2x},\rho_{2y}\right)\\
&=\frac{1}{(\lambda B)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W_{0ef}(x_1,y_1,x_2,y_2)\\
&\times\exp\left[-\frac{ikA^*}{2B^*}\left(x_1^2+y_1^2\right)+\frac{ik}{B^*}\left(x_1\rho_{1x}+y_1\rho_{1y}\right)\right]\\
&\times\exp\left[\frac{ikA}{2B}\left(x_2^2+y_2^2\right)-\frac{ik}{B}\left(x_2\rho_{2x}+y_2\rho_{2y}\right)\right]\\
&\times\exp\left[-\frac{ikD^*}{2B^*}\left(\rho_{1x}^2+\rho_{1y}^2\right)+\frac{ikD}{2B}\left(\rho_{2x}^2+\rho_{2y}^2\right)\right]dx_1dx_2dy_1dy_2,
\end{aligned}
\tag{1}
$$

where $k=2\pi/\lambda$ is the wave number with λ being the wavelength of the beam, $e\in[1-E/2,\ E/2]$and$f\in[1-F/2,\ F/2]$. $(\rho_{1x},\rho_{1y})$and $(\rho_{2x},\rho_{2y})$are two arbitrary transverse position coordinates in the output plane, (x_1,y_1) and (x_2,y_2) are two arbitrary transverse position coordinates in the source plane. A, B, C and D are the elements of a transfer matrix for the paraxial optical system.

Because the partially coherent beam array is constructed through incoherent superposition of a series of off-axis partially coherent beams, its irradiance distribution can be expressed as

$$
I(\rho_x,\rho_y)=\sum_{1-E/2}^{E/2}\sum_{1-F/2}^{F/2}W_{ef}(\rho_x,\rho_y,\rho_x,\rho_y). \tag{2}
$$

The cross-spectral density function of an off-axis circular MGSM beam at $z=0$ can be expressed as [22]

$$
\begin{aligned}
&W_{0ef}^{(c)}(x_1,y_1,x_2,y_2)\\
&=\exp\left[-\frac{(x_1-x_{0e})^2+(x_2-x_{0e})^2+\left(y_1-y_{0f}\right)^2+\left(y_2-y_{0f}\right)^2}{4\sigma^2}\right]\\
&\times\frac{1}{C_0}\sum_{m=1}^{M}\frac{(-1)^{m-1}}{m}\binom{M}{m}\exp\left[-\frac{(x_1-x_2)^2+(y_1-y_2)^2}{2m\delta^2}\right],
\end{aligned}
\tag{3}
$$

where $x_{0e}=(e-1/2)x_0$, $y_{0f}=(f-1/2)y_0$, $C_0=\sum_{m=1}^{M}\frac{(-1)^{m-1}}{m}\binom{M}{m}$ denotes a normalization factor with M being the beam index, $\binom{M}{m}$ is a binomial coefficient, σ denotes the transverse beam width, and δ is the transverse coherence width.

Substituting Eq. (3) into Eq. (1), we obtain the following expression for the cross-spectral density function of an off-axis circular MGSM beam with e and f subscripts propagating through a stigmatic ABCD optical system

$$
\begin{aligned}
&W_{ef}^{(c)}\left(\rho_{1x},\rho_{1y},\rho_{2x},\rho_{2y}\right)\\
&=\exp\left[\frac{ikD\left(\rho_{2x}^2+\rho_{2y}^2\right)}{2B}-\frac{ikD^*\left(\rho_{1x}^2+\rho_{1y}^2\right)}{2B^*}\right]\exp\left(-\frac{{x_{0e}}^2+y_{0f}^2}{2\sigma^2}\right)\\
&\times\frac{1}{(\lambda B)^2}\frac{1}{C_0}\sum_{m=1}^{M}\frac{(-1)^{m-1}}{m}\binom{M}{m}\frac{\pi^2}{M_1M_2}\\
&\times\exp\left\{\frac{1}{4M_1}\left[\left(\frac{x_{0e}}{2\sigma^2}-\frac{ik\rho_{2x}}{B}\right)^2+\left(\frac{y_{0f}}{2\sigma^2}-\frac{ik\rho_{2y}}{B}\right)^2\right]\right\}\\
&\times\exp\left[\frac{1}{4M_2}\left(\frac{ik\rho_{1x}}{B^*}+\frac{x_{0e}}{2\sigma^2}+\frac{1}{2M_1m\delta^2}\left(\frac{x_{0e}}{2\sigma^2}-\frac{ik\rho_{2x}}{B}\right)\right)^2\right]\\
&\times\exp\left[\frac{1}{4M_2}\left(\frac{ik\rho_{1y}}{B^*}+\frac{y_{0f}}{2\sigma^2}+\frac{1}{2M_1m\delta^2}\left(\frac{y_{0f}}{2\sigma^2}-\frac{ik\rho_{2y}}{B}\right)\right)^2\right],
\end{aligned}
\tag{4}
$$

where

$$
\begin{aligned}
M_1&=\frac{1}{4\sigma^2}+\frac{1}{2m\delta^2}-\frac{ikA}{2B},\quad M_2=\frac{1}{4\sigma^2}+\frac{1}{2m\delta^2}\\
&\quad+\frac{ikA^*}{2B^*}-\frac{1}{4M_1m^2\delta^4}.
\end{aligned}
\tag{5}
$$

The cross-spectral density functions of off-axis rectangular and elliptical MGSM beams at $z=0$ can be expressed by Eqs. (6) and (7) respectively [23,24], i.e.

$$
\begin{aligned}
&W_{0ef}^{(r)}(x_1,y_1,x_2,y_2)\\
&=\exp\left[-\frac{(x_1-x_{0e})^2+(x_2-x_{0e})^2+\left(y_1-y_{0f}\right)^2+\left(y_2-y_{0f}\right)^2}{4\sigma^2}\right]\\
&\times\frac{1}{C^2}\sum_{m=1}^{M}\frac{(-1)^{m-1}}{\sqrt{m}}\binom{M}{m}\exp\left[-\frac{(x_1-x_2)^2}{2m\delta_x^2}\right]\\
&\times\sum_{m=1}^{M}\frac{(-1)^{m-1}}{\sqrt{m}}\binom{M}{m}\exp\left[-\frac{(y_1-y_2)^2}{2m\delta_y^2}\right],
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
&W_{0ef}^{(e)}(x_1,y_1,x_2,y_2)\\
&=\exp\left[-\frac{(x_1-x_{0e})^2+(x_2-x_{0e})^2+\left(y_1-y_{0f}\right)^2+\left(y_2-y_{0f}\right)^2}{4\sigma^2}\right]
\end{aligned}
$$