Journal of Quantitative Spectroscopy & Radiative Transfer 217 (2018) 22–28

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Gaussian beam scattering by a particle above a plane surface

Huayong Zhang*, Yifang Yuan, Minquan Li, Zhixiang Huang

Key Lab of Intelligent Computing and Signal Processing, Ministry of Education, Anhui University, Hefei, Anhui 230039, PR China

A R T I C L E I N F O

Article history:
Received 20 February 2018
Revised 18 May 2018
Accepted 18 May 2018
Available online 24 May 2018

Keywords:
Scattering
Gaussian beam
Particle above a plane surface

A B S T R A C T

We derive a theory to calculate light scattering from a particle above an infinite plane surface and with Gaussian beam illumination. The incident Gaussian beam is expanded in series of cylindrical vector wave functions, and the scattered as well as internal fields in series of spherical ones. The integral representation of spherical vector wave functions over cylindrical waves is used to compute the reflection of the scattered field by the surface. By applying the boundary conditions and the projection technique, the unknown expansion coefficients of the scattered field are determined. For a localized beam model, numerical results of the normalized differential scattering cross sections are presented, and the scattering characteristics are discussed concisely.

1. Introduction

There has been an increasing interest in the study of the electromagnetic (EM) scattering by a particle above or behind a plane surface, due to a large number of possible applications including particle contamination characterization of a silicon wafer, reduction of reflection in optoelectronic devices, efficiency enhancement of optical antennas, and so on. Some investigations have been carried out to address the scattering problem of an EM plane wave by a spherical particle, based on the Mie theory and Fresnel surface reflection [1–4]. A coupled-dipole algorithm and a discrete source method have also been employed for the scattering problem [5,6]. Wriedt et al. introduced the extended boundary condition method (EBCM) or T-matrix method to deal with the EM plane wave scattering from a particle on or near a surface [7,8]. In [9], a numerical method known as the finite-difference time-domain (FDTD) technique was implemented. However, an incident Gaussian beam (focused TEM_{00} mode laser beam) is often used, which is of great importance in some practical situations. Recently, we have studied the transmission of a Gaussian beam through a uniaxial anisotropic slab by expanding the Gaussian beam in terms of the cylindrical vector wave functions (CVWFs) [10], and analyzed the arbitrarily shaped beam scattering from an anisotropic particle by following the projection procedure [11]. In this paper, a combination of such an expansion and the projection scheme enables us to investigate the Gaussian beam scattering by a particle above a plane surface.

The body of this paper proceeds as follows. Section 2 provides the theoretical procedure for the determination of the scattered fields of an incident Gaussian beam by a particle above a plane surface. In Section 3, numerical results for Gaussian beam scattering properties are presented. The work is summarized in Section 4.

2. Formulation

As illustrated in Fig. 1, a particle is attached to the Cartesian coordinate system $Oxyz$ (particle system). The axis Oz coincides with the normal to and origin O is located at a distance d from a plane interface between medium 1 (free space) and medium 2 (refractive index relative to that of free space denoted by $\tilde{n}$). A Gaussian beam propagates in the plane xOz and forms an angle ζ with respect to the axis Oz, with the middle of its beam waist O' having the Cartesian coordinates $(x_0, 0, z_0)$ in $Oxyz$. In this paper, the time-dependent part of the EM fields is assumed to be $\exp(-i\omega t)$.

The incident Gaussian beam strikes the particle either directly or after reflecting off the plane interface. As in [10] by us, the reflection and refraction of the Gaussian beam by the interface can be treated using the EM field expansions in terms of the CVWFs. Such an

* Corresponding author.
E-mail address: hyzhang0905@163.com (H. Zhang).

https://doi.org/10.1016/j.jqsrt.2018.05.018
0022-4073/

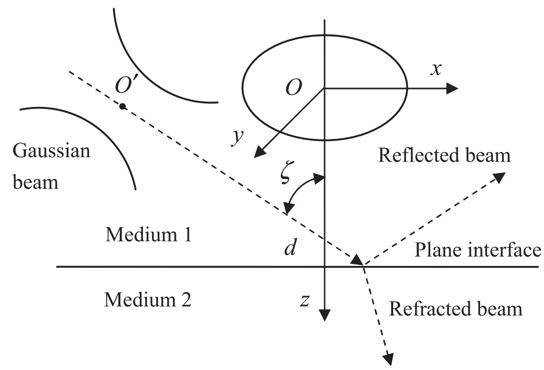

Fig. 1. Geometry of the scattering system.

expansion for the electric field of the incident Gaussian beam $\mathbf{E}^i = \mathbf{E}^{i1} + \mathbf{E}^{i2}$ with respect to the particle system $Oxyz$ is described by

$$\mathbf{E}^{i1} = E_0 \sum_{m=-\infty}^{\infty} \int_0^{\frac{\pi}{2}} [I_{m,TE}\mathbf{m}_{m\lambda}^{(1)} + I_{m,TM}\mathbf{n}_{m\lambda}^{(1)}] e^{ihz} d\alpha \tag{1}$$

and for $\mathbf{E}^{i2}$ by the same expression as $\mathbf{E}^{i1}$ but integrated over α from $\pi/2$ to π.

In Eq. (1), $\lambda = k\sin\alpha$, $h = k\cos\alpha$, k and η are respectively the free space wave number and wave impedance, and $I_{n,TE}^m$, $I_{n,TM}^m$ are the Gaussian beam shape coefficients. For a TE polarized Gaussian beam, the coefficients $I_{n,TE}^m$ and $I_{n,TM}^m$ are expressed by [10,12]

$$I_{m,TE} = \frac{i^{m+1}}{2k} \sum_{n=|m|}^{\infty} g_n \frac{2n+1}{n(n+1)} \frac{(n-m)!}{(n+m)!} \left[\frac{dP_n^m(\cos\zeta)}{d\zeta} \frac{dP_n^m(\cos\alpha)}{d\alpha} + m^2 \frac{P_n^m(\cos\zeta)}{\sin\zeta} \frac{P_n^m(\cos\alpha)}{\sin\alpha} \right] \tag{2}$$

$$I_{m,TM} = \frac{i^{m+1}}{2k} m \sum_{n=|m|}^{\infty} g_n \frac{2n+1}{n(n+1)} \frac{(n-m)!}{(n+m)!} \left[\frac{dP_n^m(\cos\zeta)}{d\zeta} \frac{P_n^m(\cos\alpha)}{\sin\alpha} + \frac{P_n^m(\cos\zeta)}{\sin\zeta} \frac{dP_n^m(\cos\alpha)}{d\alpha} \right] \tag{3}$$

where g_n (Gaussian beam shape coefficients in spherical coordinates), when the Davis model of the Gaussian beam is used [13], can be described by a simpler expression known as the original localized approximation, in the following form [14,15]

$$g_n = \frac{1}{1 + 2isl_0/w_0} \exp(ikl_0) \exp[ih(x_0 \cot\zeta - z_0)] \exp\left[\frac{-s^2(n+1/2)^2}{1 + 2isl_0/w_0} \right] \tag{4}$$

where $s = 1/(kw_0)$, $l_0 = -x_0/\sin\zeta$, and w_0 is the beam waist radius.

For a TM polarized Gaussian beam, the corresponding expansion is obtained only by replacing $I_{m,TE}$ in Eq. (1) by $-iI_{m,TM}$, and $I_{m,TM}$ by $-iI_{m,TE}$, respectively.

It should be noted that the incident TE polarized Gaussian beam reduces to a perpendicularly or s-polarized plane wave, and that the TM polarized Gaussian beam to a parallelly or p-polarized plane wave, as the beam waist $w_0 \to \infty$. As by a detailed discussion in [12], the coefficients g_n in Eq. (4) become equal to unit, by setting $w_0 \to \infty$ and $x_0 = z_0 = 0$, and then the series expansions of the s and p-polarized plane waves in terms of the CVWFs can be readily obtained from Eq. (1), described by $\mathbf{E}^i = E_0 \sum_{m=-\infty}^{\infty} \frac{i^{m+1}}{k\sin\zeta} \mathbf{m}_{m\lambda} e^{ihz}$ (electric field) and $\mathbf{H}^i = -i\frac{E_0}{\eta} \sum_{m=-\infty}^{\infty} \frac{i^m}{k\sin\zeta} \mathbf{m}_{m\lambda} e^{ihz}$ (magnetic field), respectively.

By expanding the reflected and refracted beams in series of appropriate CVWFs, and then by applying the continuous EM boundary conditions over the interface [10], as in studying the well-known Fresnel surface reflection the series expansions of the reflected beam in terms of the CVWFs can be obtained, as follows:

$$\mathbf{E}^r = E_0 \sum_{m=-\infty}^{\infty} \int_0^{\frac{\pi}{2}} [I_{m,TE} R_{TE}(\alpha) \mathbf{m}_{m\lambda}^{(1)}(-h) + I_{m,TM} R_{TM}(\alpha) \mathbf{n}_{m\lambda}^{(1)}(-h)] e^{-ihz} d\alpha \tag{5}$$

where $R_{TE}(\alpha) = R_\perp(\alpha) e^{i2hd}$, $R_{TM}(\alpha) = R_{//}\alpha) e^{i2hd}$, and $R_\perp(\alpha)$, $R_{//}(\alpha)$ respectively denote the Fresnel reflection coefficients for the perpendicular and parallel polarizations.

The electric fields scattered by and existing within the particle can be respectively expanded in terms of radiating and regular SVWFs, as follows:

$$\mathbf{E}^s = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [\alpha_{mn} \mathbf{M}_{mn}^{(3)}(k) + \beta_{mn} \mathbf{N}_{mn}^{(3)}(k)] \tag{6}$$

$$\mathbf{E}^w = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} [\delta_{mn} \mathbf{M}_{mn}^{(1)}(k') + \gamma_{mn} \mathbf{N}_{mn}^{(1)}(k')] \tag{7}$$

where α_{mn}, β_{mn}, δ_{mn} and γ_{mn} are the expansion coefficients, $k' = k\tilde{n}'$, and $\tilde{n}'$ is the refractive index of the material of the particle relative to that of free space.