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Negative extinction in one-dimensional scattering

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The energy/sec transmitted and reflected by a flat dielectric slab embedded in an absorbing host medium was calculated for normal plane wave incidence. It was found that the outgoing scattered energy/sec differs from the incoming energy/sec carried by the incident beam. This apparent energy nonconservation is compensated exactly by the energy/sec of the standing wave interference of the incoming plane wave with the outgoing reflected wave evaluated at the surface of the slab. This interference energy/sec, also known as extinction for this one-dimensional scattering situation, oscillates between positive and negative values as a function of slab thickness, in analogy to a similar effect recently predicted for scattering by a dielectric sphere embedded in an absorbing host medium.

1. Introduction

Over the last few decades, scattering of light by a dielectric sphere embedded in an absorbing host medium has attracted considerable attention [1–16]. In particular, there has been much discussion concerning the relative merits and physical interpretation of a number of different definitions of extinction for this system. These approaches differ in that (i) the incident and scattered fields are evaluated in the scattering far-zone, (often called apparent extinction) [1,3,9,11], (ii) or they are evaluated on the particle surface, (often called inherent extinction) [5,6,8,9], (iii) or they make use of the optical theorem [17] generalized to scattering in absorbing host medium [2,13,18]. Extinction has been considered as a single measurement made with the scattering particle present [1,6,8], or as the difference between two measurements made with and without the particle present [2,3,5,13–18]. Some past studies have argued that in certain regions of parameter space the extinction cross section can be less than the scattering cross section, or even negative [1,3,9,14,16]. There has been some discussion as to whether such a result is physical or not, and if it is, what interpretation can be given to it.

This study attempts to answer this question by considering a much simpler scattering system that also can be interpreted as having seemingly negative extinction. Based on the many similarities between scattering of a plane wave by a spherical particle and by a flat slab ([19], pp.42,43 of [20], and [21]), the subject of this study is the interaction of a normally incident plane wave with a transversely infinite dielectric slab embedded in an absorbing host medium. This is the converse of the well-known geometry of a plane wave normally incident on a transversely infinite absorbing slab embedded in a dielectric host medium (see Section 18.5 of [22], Section 13.4.1 of [23], and Sections 14.1 and 14.2 of [24]). The system examined here has the advantages that it is one-dimensional, it is exactly analytically soluble, the scattered wave consists only of direct reflection in the backward direction, direct transmission in the forward direction, transmission and reflection following all numbers of internal reflections, and the form of the fields is exactly the same near the slab surface and far from it. In the treatment given here, the incident and scattered energy is evaluated at the slab surfaces, analogous to inherent extinction for scattering by a sphere [9].

A significant difference between the sphere and slab scattering problems is that it is standardly assumed for the sphere problem that both the incoming and scattered fields are present everywhere in space exterior to the sphere (see Section 9.2 of [25] and [26]). On the other hand, it is standardly assumed for the slab problem that the incoming beam is removed by the scattering process, and is replaced by the outgoing scattered fields (see pp.394–395 of [22], and pp.396–398 of [27]). This difference can be motivated as follows. For scattering of a plane wave or a focused Gaussian beam by a spherical particle, the beam fields and scattered fields substantially overlap with destructive interference in the scattering near-zone, producing the deep shadow of the particle (see Fig. 7.7 of [28]). But they become easily separable as a function of scattering angle in the far-zone because of their differing angular spreading due to diffraction. On the other hand, both the incident plane wave and the slab are assumed to be of infinite transverse extent. Thus there is no angular separability of the incident and transmitted waves, and they totally overlap everywhere in the half-space behind the slab. Since the sum of the two waves is observed on a close or distant viewing screen, it is sensible and practical to call

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the observed total wave “the transmitted wave” . The consequences of artificially decomposing the transmitted wave into the sum of an incident wave plus an effective transmitted wave are discussed at the end of Section 4b. In what follows, we assume the incident beam and reflected wave are present on the illuminated side of the slab, and only the transmitted wave is present on the shadowed side. This difference from that of scattering by a sphere causes certain terms in the energy balance equation that were important for the sphere problem to vanish for the slab problem. Conversely, certain terms that vanished for the sphere problem are now of central importance to the slab problem.

For scattering of a normally incident plane wave by a transversely infinite dielectric slab embedded in an absorbing medium, the energy/sec transmitted and reflected by the slab is found to alternately be either greater than or less than the energy/sec incident on it as a function of slab thickness. This apparent nonconservation of energy is exactly compensated by the energy/sec carried by the standing wave interference of the incident and reflected waves in the illuminated region of the absorbing host medium. This interference carries no energy when the host medium is a dielectric, and does so only when the host medium is absorbing. A similar incident/reflected wave interference has been noted and discussed for scattering by a sphere embedded in an absorbing medium [2,13], and in the modeling of the Van Allen radiation belts [29].

The body of this study proceeds as follows. The energy balance equation is derived in Section 2. Transmission and reflection of a normally incident plane wave at the interface between a dielectric medium and an absorbing medium is briefly reviewed in Section 3. Section 4a then briefly reviews the well-understood case of a plane wave normally incident on an absorbing slab in a dielectric host medium. The transmission and reflection amplitudes of the slab are expressed in terms of the single-interface amplitudes of Section 3. These results obtained in Section 4a are of great use in interpreting and understanding the results of Section 4b, which analyzes the situation of current interest, namely a plane wave normally incident on a dielectric slab embedded in an absorbing host medium. The total transmission and reflection amplitudes for this geometry are obtained, and the terms contributing to the energy balance are discussed. Lastly, in Section 5 the interpretation of extinction is reconsidered and extended in light of the results of Section 4b.

2. Scattering and energy conservation

The treatment of energy conservation discussed in this section is an extension of that of pp.69–70 of [20]. Consider a volume V bounded by the surface A centered on the origin of coordinates. The surface has the outward normal $\mathbf{n}$, and the volume may or may not contain sources or sinks of electromagnetic energy. If $\mathbf{E_{total}}$ and $\mathbf{B_{total}}$ are the total complex electric and magnetic fields in the vicinity of V, and μ_0 is the permeability of free space, then

$$W_{abs} = -(1/\mu_0)\mathrm{Re}\left[\int_A (\mathbf{E_{total}} \times \mathbf{B_{total}}*)\cdot \mathbf{n}\,\mathrm{d}^2r\right] \tag{1}$$

is the net electromagnetic energy/sec absorbed within the volume. The asterisk in Eq. (1) indicates complex conjugation, and the negative sign in front of the surface integral gives the convention that a sink of electromagnetic energy is considered to be positive.

If an electromagnetic beam passes through a volume containing no electromagnetic sources or sinks, such as a dielectric homogeneous medium, then

$$\mathbf{E_{total}} = \mathbf{E_{beam}} \tag{2a}$$

$$\mathbf{B_{total}} = \mathbf{B_{beam}}, \tag{2b}$$

and

$$0 = W_{beam} \tag{3}$$

where

$$W_{beam} = -(1/\mu_0)\mathrm{Re}\left[\int_A (\mathbf{E_{beam}} \times \mathbf{B_{beam}}*)\cdot \mathbf{n}\,\mathrm{d}^2r\right]. \tag{4}$$

If the beam fields are exactly known, as is the case for a plane wave, the integral can be evaluated exactly, thus verifying Eq. (3). But if the beam is known in terms of a partial wave sum of transverse electric (TE) and transverse magnetic (TM) spherical multipole waves multiplied by beam shape coefficients, the beam fields can be written as a sum of incoming and outgoing portions [30,31] that become proportional to exp(-ikr) and exp(ikr), respectively as $r\to\infty$,

$$\mathbf{E_{beam}} = \mathbf{E_{beam}}^{\mathbf{in}} + \mathbf{E_{beam}}^{\mathbf{out}} \tag{5a}$$

$$\mathbf{B_{beam}} = \mathbf{B_{beam}}^{\mathbf{in}} + \mathbf{B_{beam}}^{\mathbf{out}}. \tag{5b}$$

This gives an implicit time-domain description of beam propagation, i.e. the beam first enters the volume V, and then afterward it leaves it. With this interpretation, Eq. (3) becomes

$$0 = W_{beam} = W_{beam}{}^{in} - W_{beam}{}^{out} + W_{beam}{}^{cross} \tag{6}$$

where

$$W_{beam}{}^{in} = -(1/\mu_0)\mathrm{Re}\left[\int_A \left(\mathbf{E_{beam}}^{\mathbf{in}} \times \mathbf{B_{beam}}^{\mathbf{in}}*\right)\cdot \mathbf{n}\,\mathrm{d}^2r\right] \tag{7a}$$

$$W_{beam}{}^{out} = (1/\mu_0)\mathrm{Re}\left[\int_A \left(\mathbf{E_{beam}}^{\mathbf{out}} \times \mathbf{B_{beam}}^{\mathbf{out}}*\right)\cdot \mathbf{n}\,\mathrm{d}^2r\right] \tag{7b}$$

$$W_{beam}{}^{cross} = -(1/\mu_0)\mathrm{Re}\{\int_A [\left(\mathbf{E_{beam}}^{\mathbf{in}} \times \mathbf{B_{beam}}^{\mathbf{out}}*\right) + \left(\mathbf{E_{beam}}^{\mathbf{out}} \times \mathbf{B_{beam}}^{\mathbf{in}}*\right)]\cdot \mathbf{n}\mathrm{d}^2r\}. \tag{7c}$$

The quantity $W_{beam}{}^{in}$ is the incoming energy/sec of the beam which is positive, $W_{beam}{}^{out}$ is the outgoing energy/sec which is also positive, and $W_{beam}{}^{cross}$ is a surface integral over the standing wave interference of the incoming and outgoing portions of the beam. For a plane wave in a dielectric medium, the triple product of the incoming and outgoing beam fields and the unit normal in Eq. (7c) is zero, and for a more general beam, the triple product is purely imaginary. Both cases give $W_{beam}{}^{cross} = 0$, so that

$$W_{beam}{}^{in} = W_{beam}{}^{out} \tag{8}$$

in agreement with Eq. (3).

A particle of arbitrary shape and refractive index is now placed at the origin of coordinates in a dielectric medium. An incident electromagnetic beam is scattered by it. The total fields are now standardly taken to be

$$\mathbf{E_{total}} = \mathbf{E_{beam}} + \mathbf{E_{scatt}}^{\mathbf{out}} \tag{9a}$$

$$\mathbf{B_{total}} = \mathbf{B_{beam}} + \mathbf{B_{scat}}^{\mathbf{out}}, \tag{9b}$$

where the scattered fields are purely outgoing and the beam fields, as in Eqs. (2a) and (2b), are those that would be present in the absence of the particle. Substitution into Eq. (1) gives

$$W_{abs}{}^{particle} = W_{beam} - W_{scatt}{}^{out} + W_{extinction} \tag{10}$$

where $W_{abs}{}^{particle}$ is the energy/sec absorbed by the particle, W_{beam} was given in Eqs. (4),(6) and (7) and

$$W_{scatt}{}^{out} = (1/\mu_0)\mathrm{Re}\left[\int_A \left(\mathbf{E_{scatt}}^{\mathbf{out}} \times \mathbf{B_{scatt}}^{\mathbf{out}}*\right)\cdot \mathbf{n}\mathrm{d}^2r\right] \tag{11a}$$