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Scattering cross-section of a cylindrical conducting particle illuminated by electromagnetic plane waves near a conducting quarter-space

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ABSTRACT

The study of the electromagnetic scattering by a particle located near a perfectly conducting corner is a fundamental topic in radar imaging, communications, remote sensing and other applications. In earlier works, the analytical modeling considered the linear field and its scattering properties. Nonetheless, energy efficiencies (or cross-sections) are meaningful fundamental physical observables connected with quadratic quantities of the linear field, which reveal additional properties of the scatterer not obtained from the linear scattered field. The aim of this work is therefore directed toward developing an improved analytical formalism for the energy efficiency factors (or cross-sections) of a cylindrical conducting particle, illuminated by electromagnetic plane waves, and located near a perfectly conducting corner-space. Incident plane progressive waves with an arbitrary angle of incidence are considered in a homogeneous, non-dissipative and non-magnetic fluid. The multiple scattering effects occurring between the electrically-conducting cylindrical particle and corner are described rigorously using the modal expansion method in cylindrical coordinates, the classical method of images and the translational addition theorem. Exact closed-form series expansions are derived and numerical examples illustrate the analysis with particular emphases on the distances from the corner-space, particle size and polarization of the incident field. The results are useful to predict the energetic scattering for various applications, and can serve to validate the results of purely numerical methods in experimental design. Some analogies with the acoustical cross-section theory are also noted.

1. Introduction

Cross-sections and their associated energy efficiency factors [1–8] are key physical observables that characterize a system in which waves (mechanical/acoustical/elastic [9–20], electromagnetic/optical [21–33], quantum [34] or gravitational fields [35]) interact with a particle or an assembly of particles, in agreement with the law of energy conservation [36].

Such quadratic quantities are intrinsically connected with radiation force [37–41] and torque [11,42,43] due to the transfer of linear and angular momenta carried by the incident waves on objects, and can be used to predict the emergence of surprising effects, such as invisibility cloaking and radiation force/torque cancellations under some specific conditions [15,16,33].

In the previous investigations, analytical expressions for the cross-sections/efficiencies were developed for the cases of single and multiple particles illuminated by an incident field. Nonetheless, the mathematical equations cannot be applied in computing the related cross-sections/efficiencies when the particle is located near a boundary/edge. In this case, multiple scatterings and interactions with the boundary/surface can occur [44–49] and this effect must be taken into account using rigorous methods since the numerical prediction of the cross-sections/efficiencies allows adequate experimental design and optimal strategies to be applied and analyzed in applications in optical manipulation, opto-fluidics, radar imaging, remote sensing and other related areas of investigation.

In some cases dealing with a microfluidic channel in opto-fluidics and radar imaging applications, a particle can be located near a corner (i.e. quarter-space), and possesses intrinsic scattering properties due to the resulting reverberations. Therefore, there is a need to develop a rigorous analytical modeling that allows accurate calculation of the (quadratic) cross-sections in the presence of a corner-space.

The purpose of this analysis is therefore directed toward developing a complete analytical modeling for the local scattering, absorption and extinction cross-sections (or efficiencies) for a circular electrically conducting cylinder in a homogeneous non-magnetic medium, illuminated by plane waves of different polarizations and arbitrary incidence angle located near a perfectly-conducting corner space. In this investigation, the multipole partial-wave series

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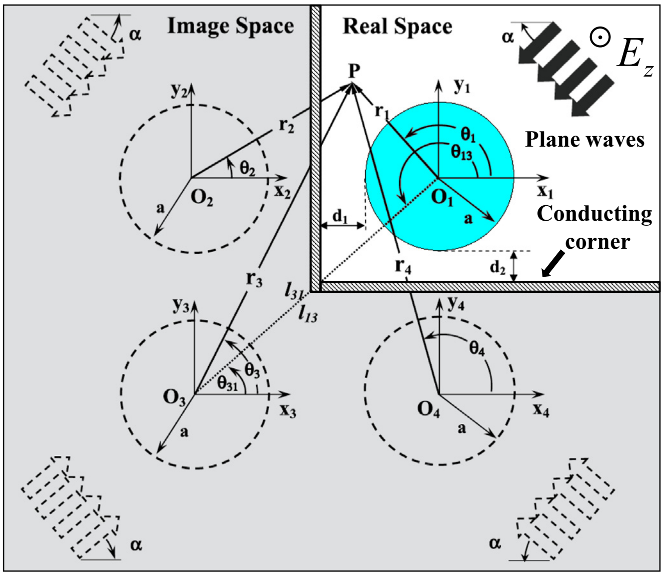

Fig. 1. Graphical representation for the interaction of an incident electromagnetic wave-field of plane progressive waves (with axial TM-polarization (i.e. along the z-axis) of the incident electric field component propagating in a homogeneous non-dissipative non-magnetic medium) with a circular conducting cylinder of radius a located near a perfectly conducting corner-space.

expansion method in cylindrical coordinates is used with the classical method of images and the translational addition theorem of cylindrical wave functions to derive exact closed-form expansions for the cross-sections. Considering the illustrative examples of a TM and TE polarizations of the primary incident wave field, numerical computations for the scattering cross-section are performed with emphases on the distances of the particle edges to the corner space, the angle of incidence, the size parameter of the cylinder and the polarization of the field. Some analogies are also noted with the field of acoustics.

In Section 2, the multipole expansion method in cylindrical coordinates is developed and exact expressions for the local cross-sections are presented. In Section 3, numerical examples are considered while Section 4 presents the conclusions.

2. Method

Consider an incident electromagnetic (EM) plane progressive waves with an axial electric field component (i.e., along the z-direction perpendicular to the polar plane, see Fig. 1) impinging on an electrically conducting cylinder of radius a located at the distances d_1 and d_2, respectively, from a perfectly conducting corner (i.e., quarter-space) as shown in Fig. 1. The medium of wave propagation is homogeneous, non-dissipative and non-magnetic with a dielectric constant ε. Since the quarter-space is perfectly conducting (i.e., the walls at right angle are perfectly conducting planes), the classical method of images [50,51] can be applied, where the original problem arrangement (i.e. cylinder + corner-space) is substituted by an equivalent configuration with four incident plane waves in an unbounded medium impinging on four analogous cylinders (Fig. 1). Therefore, the EM scattering from the cylindrical particle located near the corner can be formulated [52] as a multiple scattering phenomenon on four cylinders with four incident waves based on the multipole expansion method in cylindrical coordinates as well as the translational addition theorem of cylindrical wave functions.

The systems of coordinates and geometry are shown in Fig. 1. The origin O_1 of the cylindrical coordinate system (r_1,θ_1) is placed at the center of the object cylinder, while the origins O_p ($p=2$, 3, 4) are located at the centers of the "image" cylinders. The centers are separated by distances $l_{pj}=l_{jp}$ (p, $j=1$, 2, 3, 4) where $l_{12}=l_{34}=2(a+d_1)$ and $l_{14}=l_{23}=2(a+d_2)$. Notice that $l_{jp}=0$ if $j=p$.

Assuming a time-dependence in the form of $e^{-i\omega t}$ which has been excluded from the field equations for convenience and using the multipole expansion method [53,54], the incident real and image plane wave fields as well as their corresponding scattered fields are expressed as partial-wave series expansions of cylindrical wave functions. As a result, the j-th incident wave (corresponding to the j-th system of coordinates) in the coordinate system of the p-th cylinder is expressed by its axial steady-state (i.e. time-independent) electric field component as,

$$E_{z,inc}^{(jp)}(r_p,\theta_p)=E_0\sum_{n=-\infty}^{+\infty}\gamma_n^{(jp)}J_n(kr_p)\mathrm{e}^{in\theta_p},\tag{1}$$

where E_0 is the amplitude, $k=\varepsilon^{1/2}\omega/c$ is the wavenumber in the medium of wave propagation, ε is the dielectric permittivity, ω is the angular frequency, c is the speed of light, and $J_n(\cdot)$ is the cylindrical Bessel function of the first kind. The parameter $\gamma_n^{(jp)}=i^n e^{-in\beta_j}e^{ikl_{jp}\cos(\beta_j-\theta_{jp})}$, where $\beta_1=\pi+\alpha$, $\beta_2=-\alpha$, $\beta_3=\alpha$ and $\beta_4=\pi-\alpha$, α is the angle of incidence as shown in Fig. 1, $l_{jp}=0$ if $j=p$, and θ_{jp} is the angle between the x_j – axis and the O_jO_p line.

The corresponding incident polar magnetic field component is obtained from Faraday's law, such that [32,55–57],

$$H_{\theta,inc}^{(jp)}(r_p,\theta_p)=i\sqrt{\varepsilon}E_0\sum_{n=-\infty}^{+\infty}\gamma_n^{(jp)}J_n'(kr_p)\mathrm{e}^{in\theta_p},\tag{2}$$

where the prime denotes a derivative with respect to the argument of the cylindrical Bessel function.

The scattered axial electric field component from each cylinder in its own system of coordinates (which satisfies the Sommerfeld radiation condition) is also expressed as,

$$E_{z,\mathrm{sca}}^{(p)}(r_p,\theta_p)=E_0\sum_{n=-\infty}^{+\infty}C_n^{(p)}H_n^{(1)}(kr_p)\mathrm{e}^{in\theta_p},\tag{3}$$

where $H_n^{(1)}(\cdot)$ is the cylindrical Hankel function of the first kind of order n, and $C_n^{(p)}$ are the expansion coefficients to be determined by applying appropriate boundary conditions for the perfectly conducting cylinder.

Similarly to Eq. (2), the corresponding scattered polar component of the magnetic field from each cylinder in its own system of coordinates is expressed as,

$$H_{\theta,\mathrm{sca}}^{(p)}(r_p,\theta_p)=i\sqrt{\varepsilon}E_0\sum_{n=-\infty}^{+\infty}C_n^{(p)}H_n^{(1)'}(kr_p)\mathrm{e}^{in\theta_p}.\tag{4}$$

As noticed from Eqs. (1)–(4), the expressions for the incident and scattered electric and magnetic field components are written in different coordinate systems (Fig. 1). In order to determine the coefficients $C_n^{(p)}$, the mathematical expressions for the field components must be expressed, all referred to a common system of coordinates.

Using the translational addition theorem [58] such that [59],

$$H_n^{(1)}(kr_p)\mathrm{e}^{in\theta_p}\Big|_{r_j<l_{pj}}=\sum_{m=-\infty}^{+\infty}J_m\big(kr_j\big)H_{n-m}^{(1)}\big(kl_{pj}\big)\mathrm{e}^{i(n-m)\theta_{pj}+im\theta_j},\tag{5}$$

where $\theta_{pj}=(\pi+\theta_{jp})$, and applying the orthogonality of the complex exponential functions lead to the mathematical expressions for the series expansions of the scattered electric and magnetic field components, expressed in any j-th system of coordinates in function of the coordinate system of the p-th cylinder as,

$$E_{z,\mathrm{sca}}^{(jp)}(r_p,\theta_p)\Big|_{j\neq p}=E_0\sum_{n=-\infty}^{+\infty}D_{nm}^{(jp)}J_n(kr_p)\mathrm{e}^{in\theta_p},\tag{6}$$