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Analytical description of lateral binding force exerted on bi-sphere induced by high-order Bessel beams

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ABSTRACT

Based on the generalized multi-particle Mie equation (GMM) and Electromagnetic Momentum (EM) theory, the lateral binding force (BF) exerted on bi-sphere induced by an arbitrary polarized high-order Bessel beam (HOBB) is investigated with particular emphasis on the half-conical angle of the wave number components and the order (or topological charge) of the beam. The illuminating HOBB with arbitrary polarization angle is described in terms of beam shape coefficients (BSCs) within the framework of generalized Lorenz-Mie theories (GLMT). Utilizing the vector addition theorem of the spherical vector wave functions (SVWFs), the interactive scattering coefficients are derived through the continuous boundary conditions on which the interaction of the bi-sphere is considered. Numerical effects of various parameters such as beam polarization angles, incident wavelengths, particle sizes, material losses and the refractive index, including the cases of weak, moderate, and strong than the surrounding medium are numerically analyzed in detail. The observed dependence of the separation of optically bound particles on the incidence of HOBB is in agreement with earlier theoretical prediction. Accurate investigation of BF induced by HOBB could provide an effective test for further research on BF between more complex particles, which plays an important role in using optical manipulation on particle self-assembly.

1. Introduction

Optical binding denotes a significant phenomenon of light-matter interaction which can lead to the self-arrangement of particles into optically conjunct states [1–3]. This intriguing self-arrangement is based upon the delicate equilibrium between the optical forces resulting both from the incident beam and from the light re-scattered by the other objects. Accurate prediction of the optical binding force (BF) enables better understanding of the physical mechanism of self-organization, and may offer important applications towards contact-free storage of biological cells [4,5] and ion traps for quantum computing [6].

The research on BF was first demonstrated experimentally in early 1989s, Burns et al [7–9] found a series of bound states for two polystyrene particles and produced a stream of remarkable work that laid the foundations for the field. After that, different approaches have been developed for the theoretical prediction of BF exerted on bi-sphere system. Geometrical optics (GO) [10,11] can be employed to the prediction of BF in the ray optics regime, which requires the particles be much larger than the incident wavelength i.e. $d \gg 10\lambda$ with d the diameter of particles and λ the wavelength. Conversely, the Rayleigh dipole approximation (RDA) [12,13] can be employed to the prediction of BF exerted on particles which are much smaller than the wavelength, i.e. $d \ll \lambda$. For particles whose sizes are of incident wavelength order, both GO and RDA are inapplicable, because diffraction phenomenon cannot be neglected in this case. In order to cover the whole d/λ range, some researchers have been devoted to the rigorous prediction of BF stemming from the solution of Maxwell's equations which is suitable for modeling arbitrary number of particles of arbitrary sizes without additional approximation. For example, based on the Lorenz Mie theory [14] and the Maxwell Stress Tensor approach, Jack investigated BF between bi-sphere cluster of arbitrary size under the illumination of a plane wave [15,16]. Xu introduced the additional theorem to explore the interaction of collective homogeneous spheres [17,18]. Besides, the GMM equation between an incident beam with arbitrary profile and an assembly of spheres had already been given by Gouesbet et al. [19,20], with taking advantage of many ingredients developed for the spherical GLMT [21–23]. Following this work, Xu and Kall [24,25] put forward the extended Mie theory to calculate BF between closely spaced silver nanoparticle aggregates. Chvatal et al. presented the binding self-arrangement of a pair of Au particles in a wide Gaussian standing

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wave [26]. Despite the great wealth of knowledge obtained from these works, the investigations on BF in the previous studies are mainly focused on plane wave or Gaussian beam incidence.

Recently, due to the special characteristics of non-diffraction and self-reconstruction, Bessel beams have attracted growing attention since its naissance by Durnin [27] and have been widely applied in various fields, including optical entrapment and manipulation, optical acceleration, particle sizing and nonlinear optics [28–31]. Motivated by the features and applications of such a beam, analytical and numerical analysis are undertaken to investigate the beam expansion, field description, scattering and radiation in acoustic, optics, and microwave. Accurate description of a Bessel beam can be obtained by the beam shape coefficients as a double quadrature over spherical coordinates [32], which is the original method used in the GLMTs [33,34]. In order to overcome the time-consuming and complexity in the numerical calculation [35–37], many researchers have devoted to the analytical description of BSCs. Lock [38] analyzed the BSCs of general zero-order Bessel beams based on the angular spectrum representation (ASR). The similar procedure was also extended by Ma et al. [39] to investigate the scattering of un-polarized HOBB by spheres. Besides, Gouesbet and Lock [40,41] established the dark theorem in terms of BSCs and predicted the existence of high-order non-vortex Bessel beams. Wang derived the general description of circularly symmetric Bessel beams of arbitrary order [42,43]. Based on these representations, the scattering problem of Bessel beams by a dielectric sphere [44,45], a uniaxial anisotropic sphere [46] as well as a concentric sphere [47] has been investigated extensively by using the analytical approach. In addition, some studies have also been carried out on the trapping force induced by Bessel beams using the Rayleigh model [48], the geometrical optics [49,50] or the rigorous electromagnetic theory [51–53]. Nevertheless, the published work to which we have referred mainly focused on cases of single spherical particle. Manipulation of multiple particles simultaneously is both very different and much less mastered than that of singular sphere. Accurate prediction of BF induced by HOBB with arbitrary polarization angle is of great help for the efficient generation of optical manipulation system operating with non-diffracting beam.

In this paper, we will rely on the general description of HOBB derived by Wang et al. [54], who succeeds in dealing analytically with BSCs by using quadrature expressions in the classical framework of GLMT [21,55], with using GMM equations and EM theories to analyze lateral BF exerted on bi-sphere induced by HOBB in detail. The remainder of this paper is organized as follows. In Section 2, two kinds of descriptions on the profile of a HOBB are given. Moreover, the expansion expression and coefficients of the arbitrary polarized incident field in terms of SVWFs are given within the framework of GLMTs. Based on the GMM equation, Section 3 derives the analytical solutions to the scattering problem of a HOBB by two homogeneous spherical particles. Section 4 investigates the theoretical expressions of lateral BF between two homogeneous spheres induced by a HOBB using the EM theory. Section 5 establishes the discussions for numerical effects of various parameters and comparisons of our numerical results with earlier theoretical prediction. Finally, a conclusion is shown in Section 6.

2. Theoretical analysis

A Cartesian coordinate system $Oxyz$ is built with a fixed global coordinate system to indicate the randomness of the polarized direction of incident Bessel beam and the configuration of the bi-sphere system [Fig. 1(a)]. Considering two homogeneous spheres with radius $a_j (j = 1, 2)$ and refractive index $n_j (j = 1, 2)$ embedded in the dielectric medium with refractive index n_m. The particle co-

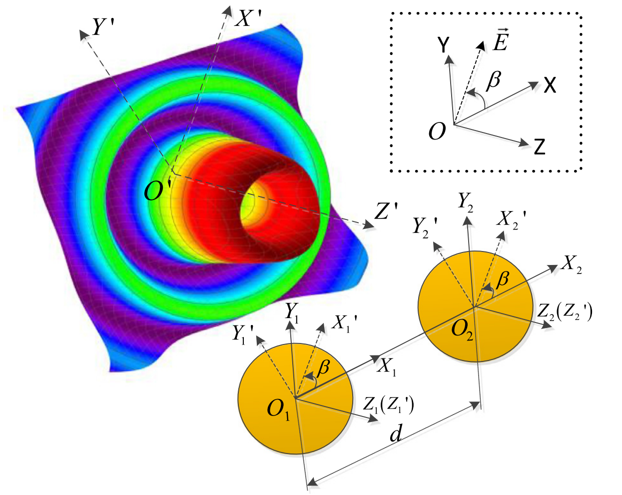

Fig. 1. Configuration of bi-sphere induced by a HOBB.d: Inter-particle distance. θ: Angle between the polarization direction and bi-sphere orientation. The set displaying is the intensity of a first-order x-polarized Bessel beam.

ordinate system $O_jx_jy_jz_j$ is established parallel to the primary system $Oxyz$, and the center of the jth sphere O_j is located at (x_j, y_j, z_j). Without loss of generality, the bi-sphere central line is along x axis and the inter-particle distance is denoted by d. The particles are vertically illuminated by a polarized HOBB that propagates in the z'-direction in the Cartesian coordinate system $O'x'y'z'$, which is known as the beam coordinate system. The coordinates of beam center O'in $Oxyz$ are (x', y', z'), and the angle between the polarization direction of HOBB and the bi-sphere central line is represented by β. This pseudo-polarization angle can then determine the polarization mode of the wave and may be regarded as a real polarization angle. For the vertical incidence, the beam is in transverse magnetic mode (TM) when $\beta = 0^0$, which corresponds to the case in which the electric vector vibrates in the incident plane (i.e. the $x'O'z'$-plane). Then, the beam is in the transverse electric mode (TE) when $\beta = 90^0$, which corresponds to the case where the magnetic vector vibrates in the incident plane (i.e. the $y'O'z'$-plane). When β presents other values, it represents another polarization mode.

The Bessel beam is assumed to propagate in an isotropic homogeneous medium and is scattered by a bi-sphere system. Electromagnetic fields outside and inside the particles must satisfy these vector wave equations (or Helmholtz equations):

$$\nabla^2\vec{E} + k^2\vec{E} = 0,\ \nabla^2\vec{H} + k^2\vec{H} = 0 \tag{1}$$

where k is the wavenumber. The solutions can be derived by introducing the SVWFs, whose expressions used here are the same as those used in Ref. [56] Eqs. (1) and ((2) there).

$$\vec{M}_{mn}^{(l)}(kr,\theta,\phi) = (-1)^m\left[im\pi_n^m(\cos\theta)\hat{i}_\theta - \tau_n^m(\cos\theta)\hat{i}_\phi\right]z_n^{(l)}(kr)e^{im\phi}$$

$$\vec{N}_{mn}^{(1)}(kr,\theta,\phi) = (-1)^m\left[\frac{n(n+1)}{kr}z_n^{(l)}(kr)P_n^m(\cos\theta)\hat{i}_r + \frac{1}{kr}\frac{d\left[rz_n^{(l)}(kr)\right]}{dr}\right]\left[\tau_n^m(\cos\theta)\hat{i}_\theta + im\pi_n^m(\cos\theta)\hat{i}_\phi\right]e^{im\phi} \tag{2}$$

where $z_n^{(l)}(kr)$ represents an appropriate kind of spherical Bessel functions: the first kind j_n, the second kind y_n, or the third kind $h_n^{(1)}$ and $h_n^{(2)}$, denoted by $l = 1$, 2, 3 or 4 respectively. $P_n^m(\cos\theta)$ is the associated Legendre Function of the first kind. Then the incident, scattered, and internal fields can be expressed as an infinite