Journal of Quantitative Spectroscopy & Radiative Transfer 214 (2018) 18–32

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

The stochastic Beer–Lambert–Bouguer law for discontinuous vegetation canopies

N. Shabanov[a,*], J.-P. Gastellu-Etchegorry[b]

[a] Institute of Space Research, Russian Academy of Sciences, Moscow, Russia
[b] CESBIO Toulouse University (CNRS, IRD, CNES, PSU), Toulouse, France

A R T I C L E I N F O

Article history:
Received 11 December 2017
Revised 19 April 2018
Accepted 19 April 2018
Available online 22 April 2018

Keywords:
Beer–Lambert–Bouguer law
3D vegetation canopy structure and density
Stochastic RT
Clumping index
Gap fraction
Vegetation fraction

A B S T R A C T

The 3D distribution of canopy foliage affects the radiation regime and retrievals of canopy biophysical parameters. The gap fraction is one primary indicator of a canopy structure. Historically the Beer–Lambert–Bouguer law and the linear mixture model have served as a basis for multiple technologies for retrievals of the gap (or vegetation) fraction and Leaf Area Index (LAI). The Beer–Lambert–Bouguer law is a form of the Radiative Transfer (RT) equation for homogeneous canopies, which was later adjusted for a correlation between fitoelements using concept of the clumping index. The Stochastic Radiative Transfer (SRT) approach has been developed specifically for heterogeneous canopies, however the approach lacks a proper model of the vegetation fraction. This study is focused on the implementation of the stochastic version of the Beer–Lambert–Bouguer law for heterogeneous canopies, featuring the following principles: 1) two mechanisms perform photon transport- transmission through the turbid medium of foliage crowns and direct streaming through canopy gaps, 2) the radiation field is influenced by a canopy structure (quantified by the statistical moments of a canopy structure) and a foliage density (quantified by the gap fraction as a function of LAI), 3) the notions of canopy transmittance and gap fraction are distinct. The derived stochastic Beer–Lambert–Bouguer law is consistent with the Geometrical Optical and Radiative Transfer (GORT) derivations. Analytical and numerical analysis of the stochastic Beer–Lambert–Bouguer law presented in this study provides the basis to reformulate widely used technologies for retrievals of the gap fraction and LAI from ground and satellite radiation measurements.

1. Introduction

An implementation of the 3D distribution of canopy foliage (a structure and density) is an essential part in development of canopy radiation models as clustering of fitoelements modifies the radiation regime and influences retrievals of canopy biophysical parameters [1–4]. Chen and Leblank [5] have identified four scales of arrangements in the architecture of the needle leaf forest stand- (1) grouping of trees on a landscape (due to succession, fire events, biodiversity, soil types variations, landscape slope, etc.), (2) grouping of foliage into tree crowns, (3) grouping of shoots into branches (4) grouping of needles into shoots. Stenberg and Smolander [6] theoretically analyzed *nesting of scales* in their study of radiative scaling of optical properties from needles to shoots to tree crowns.

One directly observable by a human eye effect of influence of a canopy structure on the radiation regime - is a streaming of sunlight through gaps in canopies. To quantify this phenomenon the notion of *directional gap fraction* [7] or often called simply *gap fraction* [8,9] has been introduced. Historically, the Beer–Lambert–Bouguer law [10] has been used to model the uncollided photons transmission through a vegetation medium, with the optical thickness quantified by Leaf Area Index (LAI). Multiple ground-based technologies have been implemented to measure radiation below (or above) canopy and utilized the Beer–Lambert–Bouguer law to infer the directional gap fraction and LAI. The examples include AccuPAR (Decagon Devices, Pullman, WA, USA, www.decagon.com), SanScan (Delta-T Devices Ltd, Cambridge, UK, www.delta-t.co.uk), LAI-2000 (LI-COR, Linkoln, NE, USA, www.licor.com), DEMON (CSIRO, Canbera, Australia, www.cbr.clw.csiro.au/pyelab/tour/demon.htm) instruments and the associated technologies, and also the Digital Hemispherical Photography (DHP) measurements processed by various software packages, such as CAN_EYE (http://www.avignon.inra.fr/can_eye). A comprehensive review of performance of various techniques is provided by Breda [11]. Satellite data-based technologies were also implemented to derive vegetation fraction applying the linear mixture model to Vegetation Indices (VI_s) [12,13].

* Corresponding author.
E-mail address: nikolay.shabanov@d902.iki.rssi.ru (N. Shabanov).

https://doi.org/10.1016/j.jqsrt.2018.04.021
0022-4073/

In spite of a proven practical feasibility of the above technologies, they face theoretical controversies. The Beer–Lambert–Bouguer law is a form of the Radiative Transfer (RT) equation, describing propagation of uncollided photons in the homogeneous media and is not suitable to describe streaming of light through gaps in the heterogeneous vegetation. Even though a correction coefficient (the clumping index) for the Beer–Lambert–Bouguer law has been developed [9], the equation remained to be applicable to homogeneous media only, as shown in this paper. Further, the empirical linear mixture model has been utilized to model relationship between VIs and the vegetation fraction has encountered a controversy concerning validity of the linear assumption [12].

The solution of those theoretical issues is available in a framework of the Stochastic Radiative Transfer (SRT) theory [1,14,15]. This radiation model was formulated particularly for heterogeneous vegetation canopies, and features a mechanism of streaming radiation through canopy gaps. The SRT is the 1D equation, where heterogeneity of media is expressed in terms of the statistical moments of a vegetation structure. While the model has been implemented, it still misses a critical element- a physically based, consistent with measurements model of the vegetation fraction as a function of LAI.

In this paper we reviewed performance of several well-known semi-empirical approaches for retrieval of the gap fraction from radiation measurements, identified controversies and demonstrated utility of the Stochastic RT to reproduce features of the radiation field in heterogeneous canopies missing in the traditional approaches. This study is focused on the implementation of the stochastic version of the Beer–Lambert–Bouguer law for the heterogeneous canopy, featuring correlation of a canopy structure. A proper model of the gap fraction provides the missing core element of canopy model for the end-to-end implementation of the full stochastic RT model.

The theoretical results obtained in this paper provide the basis to reformulate widely used techniques for retrievals of the gap (vegetation) fraction and LAI from ground and satellite radiation measurements. This paper is organized as follows. In Section 2 we review several relevant semi-empirical approaches for modeling the gap fraction. In Section 3 we describe basic concepts of the Stochastic RT model. In Section 4 we formulate the theoretical basis of the stochastic Beer–Lambert–Bouguer law. Section 5 provides a numerical analysis in support of the theoretical basis developed in the former section. Summary of key theoretical findings and outlook for the future model development concludes this paper.

2. Relevant semi-empirical approaches for modeling the vegetation fraction for ground- and satellite-data based applications

According to a widely used semi-empirical approach, transmittance of the direct solar radiation is modeled by the Beer–Lambert–Bouguer law,

$$t(LAI, \vec{\Omega}) = \exp(-\gamma \cdot G(\vec{\Omega}) \cdot LAI/\mu(\vec{\Omega})), \qquad (1)$$

where $\vec{\Omega}$ is the direction of observation, LAI is the Leaf Area Index- one-sided leaf area per unit ground area, $G(\vec{\Omega})$- is the mean projection of leaf normal on the direction $\vec{\Omega}$, $\mu(\vec{\Omega}) \equiv \cos(\vec{\Omega})$, and γ is the *clumping index*, explained below. When $\gamma = 1$, Eq. (1) collapses to the standard expression for the uncollided photons transmittance in the turbid medium, derived according to the RT principles [16]. In attempt to apply this equation to the heterogeneous canopy the equation was reformulated using the statistical principles [9,17]. Canopy transmittance was interpreted as the canopy directional gap fraction. When fitoelements are distributed according to the Poison law (random and statistically independent) the gap fraction is described by Eq. (1) with $\gamma = 1$. However, when fitoelements are correlated (with the correlation coefficient γ), the Poisson statistics is replaced by the Markov chain theory, resulting in Eq. (1) for an arbitrary value of γ. A technology has been implemented to measure the clumping index on ground using TRAC instrument (Third-Wave Engineering, Ottawa, Canada, www.ccrs.nrcan.gc.ca/ccrs/tekrd/rd/apps/em/) or processing DHP measurements. An algorithm has also been implemented to generate the clumping index from space measurements, such as multi-angular POLDER data [18].

The Beer–Lambert–Bouguer law (with clumping) mostly has been utilized to analyze the ground radiation measurements to infer the gap fraction and an *effective* LAI ($= \gamma \cdot LAI$) using AccuPAR, SunScan, LAI-2000, DEMON [11] and the Digital Hemispherical Photography (DHP) [19] technologies. The *effective* LAI can be converted to the true LAI using an estimate of the clumping index according to the adopted methodology (Eq. (1)).

However, the original Beer–Lambert–Bouguer law exhibits multiple theoretical controversies. Using the stochastic RT principles (cf. Section 4.3) we will demonstrate, that introducing clumping index does not change the scope of applicability of the equation- it still remains an equation for the turbid medium. Also, consider the following cases. May the canopy transmittance modeled by Eq. (1) be interpreted as the gap fraction? Consider two canopy stands modeled by cylinders, first stand two times shorter than the other (other parameters being the same). LAI in the first case is two times smaller, therefore canopy transmittance is higher according to the Beer–Lambert–Bouguer law, but the gap fraction is the same for both. Next, consider a canopy stand, where foliage is packed in the distinct crowns with high LAI density, but crowns are separated by open space. LAI of such canopy is high. According to the Beer–Lambert–Bouguer law (with clumping) canopy transmittance should approach zero, however light should be able to stream directly to the ground through open space between crowns.

The second line of research involves the retrievals of the vegetation fraction from satellite measurements of spectral reflectances or the Vegetation Indices (VI_s) using empirical linear (or nonlinear) mixture models [20]. The vegetation fraction is defined as a fractional area of the vegetation occupying each grid cell (the horizontal density). According to the linear model, two (or more) end members are defined: pure dense closed canopy (corresponding to the vegetation fraction equal one), and pure soil (the vegetation fraction is zero). The end-members states are characterized by the corresponding values of VI. VI at any intermediate case is modeled as a linear combination of the end-member's VI with weights expressed through the vegetation fraction,

$$VI = p \cdot VI_V + (1 - p) \cdot VI_S \rightarrow p = \frac{VI - VI_S}{VI_V - VI_S}. \qquad (2)$$

Gutman and Ignatov [21] applied a linear mixture model for the Normalized Difference Vegetation Index (NDVI) derived from the Advanced Very High Resolution Radiometer (AVHRR) data. The NOAA Green Vegetation Fraction (GVF) product is currently generated from the Visible Infrared Imaging Radiometer Suite (VIIRS) data using the linear mixture model, but applied to the Enhanced Vegetation Index (EVI). GVF is defined as the fraction of a pixel covered by green vegetation if it were viewed vertically (NOAA VIIRS GVF Algorithm Theoretical Basis Document, http://npp.gsfc.nasa.gov/documents.html). Below we demonstrate the relationship between satellite data derived GVF, EVI, NDVI and LAI. Former three data sets are from the corresponding NOAA VIIRS GVF and Surface Reflectance products, the latter is from NASA LAI MODerate Resolution Imaging Spectroradiometer (MODIS) product (NASA VIIRS LAI was not available at the time of writing). All involved data sets were gridded to 1/20° geographic projection. Fig. 1 compares