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The standard perturbation method for infrared radiative transfer in a vertically internally inhomogeneous scattering medium

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ABSTRACT

A new scheme based on the standard perturbation method is proposed to solve the problem of infrared radiative transfer (applicable to 4–1000 µm) in a scattering medium, in which the inherent optical properties are vertically inhomogeneous. In this scheme, we use the exponential formulas to fit the vertical variation of the optical properties first, and then use the perturbation method to resolve the nonlinear equations. In the perturbation method, the standard two-stream approximation is used as the zeroth-order solution and multiple scattering effect of the continuous changing optical properties is included in the first-order solution. By applying the new solution to an idealized medium, the new solution is found well suited for solving the infrared radiative transfer in the vertically inhomogeneous medium. The errors of the standard two-stream solution can be up to 6% for upward emissivity and 3% for downward emissivity, while the errors of the new solution is limited to 2% and 0.4%, respectively. Under the different circumstances of incident radiation from the bottom, the relative errors of downward emissivity for the new solution are also generally smaller than those for the standard two-stream solution. We also apply the new solution to two cases of water cloud in an infrared band (5–8 µm) of a radiative model and at a wavelength (11 µm) of atmospheric window. The spectrums are quite suitable for studying the optical properties of clouds. In the band of 5–8 µm, the maximum relative errors of downward emissivity can reach −11% for the standard two-stream solution and is only −2% for the new solution. At the wavelength of 11 µm, the results of upward and downward emissivity for the solution are also much more accurate than those for the standard two-stream solution. The code, coupled with the inhomogeneous infrared radiative transfer solution's code, is available from the authors upon request.

1. Introduction

Finding the solution of radiative transfer equation (RTE) is a key issue in a radiation scheme for climate modeling and remote sensing. In most current radiation schemes, the atmosphere is vertically divided into multiple sub-layers. For each sub-layer, the inherent optical properties (IOPs) are fixed, assuming that each sub-layer is internally homogeneous. Based on this assumption of internal homogeneity, various approximation techniques have been proposed to solve the RTE (e.g., [6,13,16,18,19,21,28,29,34,37]).

However, observations have verified that the liquid water content (LWC), the cloud droplet size distribution, and the IOPs of cloud droplets vary with altitude [1,2,24,31,33]. Li et al. [17] developed a Monte Carlo radiative transfer model that considers the inhomogeneity of IOPs. The results showed that the differences in reflectance between the vertically inhomogeneous clouds and their homogeneous counterparts can reach 10% for large solar zenith angles. Because of its time-consuming processes, the Monte Carlo method is neither suitable for climate simulation nor for remote sensing [20]. Zhang et al. [36] developed a new RTE solution which is based on the standard perturbation method and provided an efficient way to deal with the shortwave radiative transfer in a vertically inhomogeneous medium.

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Since the scattering effect is much weaker for infrared radiation than for solar radiation, the absorption approximation (AA) is widely used in current climate models to solve the infrared radiative transfer (IRT) [27]. In AA, scattering is neglected in all but the forward direction, yielding large errors in cloudy sky cases [22,23]. Furthermore, previous studies have shown that AA can produce errors of >4 W m^{-2} at the top of atmosphere for outgoing infrared radiation in climate simulations [6,14,30].

Conversely, when the scattering effect is considered, IRT can be solved by using approximation techniques that are originally used for solving the transfer of solar radiation. The simplest and the most widely used technique for radiative transfer in climate models is the two-stream approximation [6,25], of which the accuracy have been investigated by King and Harshvardhan [11]. Similar to solar radiation, the two-stream approximation for IRT is based on the assumption of internal homogeneity, in which the variations of IOPs inside each layer are virtually ignored. Zhang et al. [35] introduced the adding method based on invariance principle to solve the layer connections in IRT for an inhomogeneous atmosphere.

In principle, IRT in a vertically inhomogeneous scattering medium can be solved by increasing the number of sub-layers of the atmosphere. However, even the most current climate model [12] only includes 30–100 sub-layers, which is not high enough to address the internal variation of IOPs. The main purpose of this study is to develop a new inhomogeneous IRT solution that can handle vertical inhomogeneity of IOPs with a limited number of sub-layers in climate model. This solution follows the standard perturbation method: the two-stream approximation for homogeneous layer is used for the zeroth-order solution and inhomogeneous scattering effect is considered in the first-order solution. In the following section, the basic theory of the new inhomogeneous IRT is introduced along with the derivation of its solution. In Section 3, the accuracy and practicality of the new inhomogeneous solution are investigated by applying the solution to an idealized medium and two cases of water cloud. A brief discussion is summarized in Section 4.

2. IRT solution of a inhomogeneous medium

In climate modeling, we are concerned about radiative flux calculations. So we begin with the azimuthally averaged infrared radiative transfer equation for intensity $I(\tau,\mu)$ in plane-parallel atmospheres:

$$\mu\frac{dI(\tau,\mu)}{d\tau}=I(\tau,\mu)-\frac{\omega(\tau)}{2}\int_{-1}^{1}I(\tau,\mu)P(\tau,\mu,\mu')d\mu'-(1-\omega(\tau))B(T), \quad (1)$$

where μ is the cosine of the zenith angle; $P(\tau,\mu,\mu')$ is the azimuthally-averaged scattering phase function where τ is the optical depth [τ is equal to 0 (τ_0) at the top (bottom) of the medium]; $\omega(\tau)$ is the single scattering albedo; and $B(T)$ is the Planck function at temperature T, which represents the internal infrared emission of the medium. Since the values of optical properties are closely related to spectrum band selection. here we discuss the infrared band (4–1000 µm). And the schematic illustration of infrared radiative transfer in a layer is shown in Fig. 1.

The Planck function is approximated linearly as a function of optical depth [30] as

$$B[T(\tau)]=B_0+\beta\tau, \quad (2)$$

where $\beta=(B_1-B_0)/\tau_0$ and τ_0 is the total optical depth of the medium. The Planck functions B_0 and B_1 are evaluated by using the temperature of the top ($\tau=0$) and the bottom ($\tau=\tau_0$) of the medium.

According to the two-stream approximation, upward and downward intensities are $I(\tau,\mu_1)=I^+(\tau)$ and $I(\tau,\mu_{-1})=I^-(\tau)$, respectively, where $\mu_1=-\mu_{-1}=1/1.66$ is a diffuse factor that converts radiative intensity to flux [4]. $\int_{-1}^{1}I(\tau,\mu)P(\tau,\mu,\mu')d\mu'$ can be written as

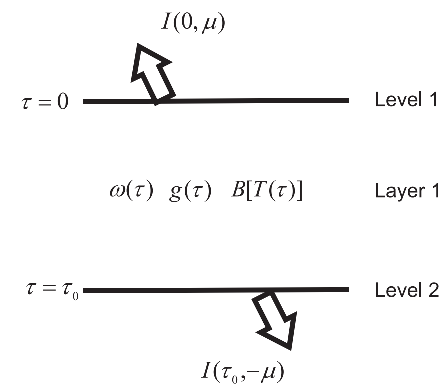

Fig. 1. The schematic illustration of infrared radiative transfer in a layer.

$$\int_{-1}^{1}I(\tau,\mu)P(\tau,\mu,\mu')d\mu'=[1+3g(\tau)\mu\mu_1]I^+(\tau)+[1+3g(\tau)\mu\mu_{-1}]I^-(\tau), \quad (3)$$

where $g(\tau)$ is the asymmetry factor.

Using Eqs. (1) and (3), we can obtain

$$\frac{dI^+(\tau)}{d\tau}=\gamma_1(\tau)I^+(\tau)-\gamma_2(\tau)I^-(\tau)-\gamma_3(\tau)B(\tau), \quad (4a)$$

$$\frac{dI^-(\tau)}{d\tau}=\gamma_2(\tau)I^+(\tau)-\gamma_1(\tau)I^-(\tau)+\gamma_3(\tau)B(\tau), \quad (4b)$$

where $\gamma_1(\tau)=\frac{1-\omega(\tau)(1+g(\tau))/2}{\mu_1}$, $\gamma_2(\tau)=\frac{\omega(\tau)(1-g(\tau))}{2\mu_1}$, and $\gamma_3(\tau)=\frac{1-\omega(\tau)}{\mu_1}$.

In climate models, the single scattering albedo $\omega(\tau)$ and asymmetry factor $g(\tau)$ are constant within each sub-layer. Therefore, the internal variation of cloud IOPs are virtually ignored and the cloud media is taken as internally homogeneous. In the following equation, we consider that $\omega(\tau)$ and $g(\tau)$ vary with optical depth. The single scattering albedo and the asymmetry factor can be expressed as

$$\omega(\tau)=\hat{\omega}+\varepsilon_\omega(e^{-a_1\tau}-e^{-a_1\tau_0/2}), \quad (5a)$$

$$g(\tau)=\hat{g}+\varepsilon_g(e^{-a_2\tau}-e^{-a_2\tau_0/2}), \quad (5b)$$

where $\hat{\omega}$ and $\hat{g}$ are the values of single scattering albedo and asymmetry factor at $\tau_0/2$, respectively. The ε_ω, ε_g, a_1, and a_2 are fitting parameters which can be obtained after fitting the exact values of $\omega(\tau)$ and $g(\tau)$. Both ε_ω and ε_g are small parameters since the internal variations of $\omega(\tau)$ and $g(\tau)$ for clouds are much smaller than $\hat{\omega}$ and $\hat{g}$ in realistic conditions. By substituting Eqs. (5a) and (5b) into $\gamma_1(\tau)$, $\gamma_2(\tau)$, and $\gamma_3(\tau)$ and by ignoring the second order of the small parameters of ε_ω^2, ε_g^2, and $\varepsilon_\omega\varepsilon_g$, we can obtain

$$\gamma_1(\tau)=\gamma_1^0+\gamma_1^1\varepsilon_\omega(e^{-a_1\tau}-e^{-a_1\tau_0/2})+\gamma_1^2\varepsilon_g(e^{-a_2\tau}-e^{-a_2\tau_0/2}), \quad (6a)$$

$$\gamma_2(\tau)=\gamma_2^0+\gamma_2^1\varepsilon_\omega(e^{-a_1\tau}-e^{-a_1\tau_0/2})+\gamma_2^2\varepsilon_g(e^{-a_2\tau}-e^{-a_2\tau_0/2}), \quad (6b)$$

$$\gamma_3(\tau)=\gamma_3^0+\gamma_3^1\varepsilon_\omega(e^{-a_1\tau}-e^{-a_1\tau_0/2}), \quad (6c)$$