# Feasibility of inverse problem solution for determination of city emission function from night sky radiance measurements 

Jaromír Petržala<br>ICA, Slovak Academy of Sciences, Dúbravská cesta, Bratislava 9, 84503, Slovakia

## A R T I C L E I N F O

## Article history:

Received 9 October 2017
Revised 9 March 2018
Accepted 16 April 2018
Available online 17 April 2018

## Keywords:

City emission function
Sky radiance
Inverse problem
Tikhonov regularization
Parameter choice
L-curve


#### Abstract

The knowledge of the emission function of a city is crucial for simulation of sky glow in its vicinity. The indirect methods to achieve this function from radiances measured over a part of the sky have been recently developed. In principle, such methods represent an ill-posed inverse problem. This paper deals with the theoretical feasibility study of various approaches to solving of given inverse problem. Particularly, it means testing of fitness of various stabilizing functionals within the Tikhonov's regularization. Further, the L-curve and generalized cross validation methods were investigated as indicators of an optimal regularization parameter. At first, we created the theoretical model for calculation of a sky spectral radiance in the form of a functional of an emission spectral radiance. Consequently, all the mentioned approaches were examined in numerical experiments with synthetical data generated for the fictitious city and loaded by random errors. The results demonstrate that the second order Tikhonov's regularization method together with regularization parameter choice by the L-curve maximum curvature criterion provide solutions which are in good agreement with the supposed model emission functions.


© 2018 Elsevier Ltd. All rights reserved.

## 1. Introduction

An estimation of an artificial sky brightness is important not only to astronomers, who need to eliminate this light pollution, but also for environmentalists [1] as well as for urban planners. In general, sky glow observed in a vicinity of a city depends on the optical properties of a local atmosphere, angular behavior of city light emissions, and also on spectral characteristics of light sources. The optics of the atmosphere is strongly influenced by the physical properties of aerosol particles spread in the air. The knowledge of them is the necessary condition for modeling of radiative transfer in the atmosphere. They can be, in principle, determined from daylight measurements, e.g. [2], and at stable conditions we can consider them as a priori known. However, our object of interest here is the function describing how a city radiates as the whole. The function that forms the first input information when we want to compute at least approximately the night sky radiance pattern in an arbitrary position near the city.

Individual light sources with various directional distribution of their emissions and with non-uniform layout over a city area, together with overall heterogenity of an urban environment, form a complex source of upward light emissions. All these partial effects coalesce into a bulk emission function determining an angu-

[^0]lar distribution of the emitted radiation. Although the radiance of diffusely reflected light is isotropic, we can suppose that the bulk direct uplight of weakly shielded street lamps, building lights, etc. will have a sharply increasing radiance at low elevation angles. Obtaining this bulk emission function by direct measurements is extremely difficult, if not impossible.

There is the possibility to retrieve the emission function indirectly from sky radiances, which may be easily and routinely measured, as a solution of the inverse problem. Such approach was applied recently in [3-5] with using of Tikhonov's regularization method. The crucial task of Tikhonov's regularization is to find an optimal value of the so called regularization parameter, which controls the trade-off between the smoothness of the solution and the data fidelity, i. e. how good the solution predicts the measured data. In the cited works, this task is solved in the sense of the discrepancy principle. However, this method requires information about the data errors and, in general, also an inaccuracy of the model should be taken into account. In practice, a close estimation of the noise level may not be known. Moreover, the optical parameters of the atmospheric environment are not known exactly what introduces further uncertainty to the model. To avoid the mentioned requirements, some other well known techniques as the L-curve method or generalized cross validation should be used. The main aim of this paper is to examine feasibility of this methods in solving of the proposed inverse problem. Together with
short investigation of what stabilizing functional would optimally represent the expected angular behavior of the emission function.

In Section 2 we present the theoretical model for calculation of diffuse sky spectral radiance in urban surroundings as a functional of the emission function. This model forms the basis of our inverse problem. Its regularized solution together with the examined techniques of regularized parameter selection are described in Section 3. In Section 4 some results of numerical experiments performed with synthetic data for a fictitious city are presented.

## 2. Theoretical model of sky radiance

Although ground based light sources can have very complex directional characteristics of radiative emissions it is obvious to simplify the problem considering the azimuthal symmetry. Hence, we suppose that an "infinitesimal" surface element of area $d A$ radiates with the spectral radiance $L_{E}\left[\mathrm{~W} . \mathrm{m}^{-2} . \mathrm{sr}^{-1} . \mathrm{nm}^{-1}\right]$ which is just a function of the zenith angle $\vartheta_{\mathrm{E}}$. The further simplification of the problem is due to the assumption of the plane-parallel model of the atmosphere. Its optical properties depend just on a height $h$ above the ground.

The radiation propagating through the atmosphere is scattered and absorbed by air gas molecules and also by aerosol particles spread in the air. Thus the volume extinction coefficient describing optical properties of the stratified atmosphere $k_{\text {ext }}(h)=k_{\text {ext }, m}(h)+k_{\text {ext, } a}(h)$ where $k_{\text {ext, }} m(h)$ and $k_{\text {ext, }}(h)$ are the corresponding coefficients for molecular and aerosol extinction. Their vertical profile is obviously approximated by the exponential altitude dependence
$k_{e x t, m}=\frac{\tau_{m}}{h_{m}} \exp \left(-\frac{h}{h_{m}}\right)$,
$k_{e x t, a}=\frac{\tau_{a}}{h_{a}} \exp \left(-\frac{h}{h_{a}}\right)$,
where $\tau_{m}$ and $\tau_{a}$ are the molecular and aerosol optical thickness of the atmosphere, $h_{m}$ and $h_{a}$ are scale heights. There are a lot of approximate relations used to calculate $\tau_{m}$ for a specific radiation wavelength (e.g. [6]). $\tau_{a}$ is the optional parameter of the model. We consider $h_{m}=8 \mathrm{~km}$ and $h_{a}=2 \mathrm{~km}$ as the typical values (see e.g. $[7,8])$. The optical thickness of the atmospheric layer ranging from the ground to the height $h$ is then

$$
\begin{align*}
\tau(h) & =\int_{0}^{h} k_{\text {ext }}(z) d z \\
& =\tau_{m}\left(1-e^{-h / h_{m}}\right)+\tau_{a}\left(1-e^{-h / h_{a}}\right) \tag{2}
\end{align*}
$$

The scattering on a particle is characterized by the scattering phase function that gives the angular distribution of the scattered radiation. In the case of air molecules it is defined by the Rayleigh scattering phase function
$p_{m}(\gamma)=\frac{3}{4}\left(1+\cos ^{2} \gamma\right)$,
where $\gamma$ is the scattering angle. In the case of aerosols the phase function depends on a particle's shape, size and internal structure. In modeling of radiative transfer it is convenient to replace this complex phase function by the simplified Henyey-Greenstein formula
$p_{a}(\gamma)=\frac{1-g^{2}}{\left(1+g^{2}-2 g \cos \gamma\right)^{3 / 2}}$,
where $g$ is the so called asymmetry parameter. This is another optional parameter which has to be specified in the model. Typically $g$ appears to be equal approximately 0.6 in the case of atmospheric aerosols [9]. The total amount of radiative energy scattered by the


Fig. 1. Configuration of the scattering problem.
atmospheric volume element to an arbitrary direction will be proportional to the quantity

$$
\begin{align*}
\Gamma(h, \gamma)= & k_{e x t, m}(h) \frac{\varpi_{m}}{4 \pi} p_{m}(\gamma) \\
& +k_{e x t, a}(h) \frac{\varpi_{a}}{4 \pi} p_{a}(\gamma) \tag{5}
\end{align*}
$$

$\varpi_{m}$ and $\omega_{a}$ are the molecular and aerosol single scattering albedo. For the molecular scatterer $\omega_{m} \approx 1$. The single scattering albedo of aerosols can vary in range from 0 to 1 , but obviously it is not smaller than 0.8 at visible light wavelengths [10]. However, $\omega_{a}$ forms the third optional model parameter.

Now consider the configuration illustrated in Fig. 1. The radiating surface element is in the position $\mathbf{R}=R(\cos \Phi,-\sin \Phi, 0)$ with regard to the observer 0 . A particular atmospheric volume element from which the scattered light comes to the observer has the position vector $\mathbf{r}=r(\cos \varphi \sin \vartheta,-\sin \varphi \sin \vartheta, \cos \vartheta)$ with regard to the observer and $\mathbf{r}_{\mathbf{E}}=r_{E}\left(\cos \varphi_{E} \sin \vartheta_{E},-\sin \varphi_{E} \sin \vartheta_{E}, \cos \vartheta_{E}\right)$ with regard to the light source. So $\mathbf{r}=\mathbf{r}_{\mathbf{E}}+\mathbf{R}$.

The spectral flux density of radiative energy incident on the volume element at the position $\mathbf{r}_{\mathbf{E}}$ with regard to the light source will be

$$
\begin{align*}
F_{E}\left(\mathbf{r}_{\mathbf{E}}\right)= & L_{E}\left(\vartheta_{E}\right) d A \frac{\cos ^{3} \vartheta_{E}}{h^{2}} \\
& \times \exp \left(-\frac{\tau(h)}{\cos \vartheta_{E}}\right) . \tag{6}
\end{align*}
$$

When we confine ourselves to the single scattering, we obtain for the spectral radiance detectable by the observer $O$ in the direction $(\vartheta, \varphi)$ the following formula

$$
\begin{align*}
I(\vartheta, \varphi) & =\int_{0}^{H} \frac{S(\mathbf{r})}{\cos \vartheta} \exp \left[-\frac{\tau(h)}{\cos \vartheta}\right] d h, \\
S(\mathbf{r}) & =\Gamma(h, \gamma) F_{E}\left(\mathbf{r}_{\mathbf{E}}\right) \tag{7}
\end{align*}
$$

where $H$ is the maximum considered height of the atmosphere. The scattering angle $\gamma$ is given by the relations

$$
\begin{align*}
\cos \gamma & =-\cos \vartheta \cos \vartheta_{E}-\sin \vartheta \sin \vartheta_{E} \cos \left(\varphi-\varphi_{E}\right), \\
\vartheta_{E} & =\arccos \left(\frac{h}{r_{E}}\right) \\
r_{E}^{2} & =\frac{h^{2}}{\cos ^{2} \vartheta}-2 R h \tan \vartheta \cos (\varphi-\Phi)+R^{2}, \\
\cos \varphi_{E} & =\frac{h \tan \vartheta \cos \varphi-R \cos \Phi}{h \tan \vartheta_{E}} \\
\sin \varphi_{E} & =\frac{h \tan \vartheta \sin \varphi-R \sin \Phi}{h \tan \vartheta_{E}} \tag{8}
\end{align*}
$$

Whence the emission function $L_{E}$ depends just on the angle $\vartheta_{E}$, we need to transform the integral with respect to $h$ in Eq. (7) into the integral with respect to this angle. This must be done carefully because the function $\vartheta_{E}(h)$ is not always monotonic. When the angle between vectors $\mathbf{r}$ and $\mathbf{R}$ is less or equal to $\pi / 2$, then $\vartheta_{E}$ at first decreases from $\pi / 2$ to some

# https://daneshyari.com/en/article/7845991 

Download Persian Version:

## https://daneshyari.com/article/7845991

## Daneshyari.com


[^0]:    E-mail address: usarjape@savba.sk
    https://doi.org/10.1016/j.jqsrt.2018.04.019 0022-4073/© 2018 Elsevier Ltd. All rights reserved.

