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Evaluation of the CPU time for solving the radiative transfer equation with high-order resolution schemes applying the normalized weighting-factor method

J. Xamán [a,*], I. Zavala-Guillén [b], I. Hernández-López [a], J. Uriarte-Flores [a], I. Hernández-Pérez [c], E.V. Macías-Melo [c], K.M. Aguilar-Castro [c]

[a] Centro Nacional de Investigación y Desarrollo Tecnológico, CENIDET-TecNM-SEP, Prol. Av. Palmira S/N. Col. Palmira, Cuernavaca, Morelos CP 62490, México
[b] Centro de Investigación Científica y de Educación Superior de Ensenada, CICESE, Carretera Ensenada-Tijuana No. 3918, Zona Playitas, Ensenada, Baja California, CP 22860, México
[c] División Académica de Ingeniería y Arquitectura DAIA-UJAT, Universidad Juárez Autónoma de Tabasco, Carretera Cunduacán-Jalpa de Méndez km. 1, Cunduacán, Tabasco, CP 86690, México

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ABSTRACT

In this paper, we evaluated the convergence rate (CPU time) of a new mathematical formulation for the numerical solution of the radiative transfer equation (RTE) with several High-Order (HO) and High-Resolution (HR) schemes. In computational fluid dynamics, this procedure is known as the Normalized Weighting-Factor (*NWF*) method and it is adopted here. The *NWF* method is used to incorporate the high-order resolution schemes in the discretized RTE. The *NWF* method is compared, in terms of computer time needed to obtain a converged solution, with the widely used deferred-correction (*DC*) technique for the calculations of a two-dimensional cavity with emitting–absorbing–scattering gray media using the discrete ordinates method. Six parameters, viz. the grid size, the order of quadrature, the absorption coefficient, the emissivity of the boundary surface, the under-relaxation factor, and the scattering albedo are considered to evaluate ten schemes. The results showed that using the *DC* method, in general, the scheme that had the lowest CPU time is the SOU. In contrast, with the results of the*DC* procedure the CPU time for DIAMOND and QUICK schemes using the *NWF* method is shown to be, between the 3.8 and 23.1% faster and 12.6 and 56.1% faster, respectively. However, the other schemes are more time consuming when the*NWF*is used instead of the *DC* method. Additionally, a second test case was presented and the results showed that depending on the problem under consideration, the *NWF* procedure may be computationally faster or slower that the *DC* method. As an example, the CPU time for QUICK and SMART schemes are 61.8 and 203.7%, respectively, slower when the *NWF* formulation is used for the second test case. Finally, future researches to explore the computational cost of the *NWF* method in more complex problems are required.

1. Introduction

From the 60's to the present time, the radiation phenomenon in participant media has been represented with the radiative transfer equation (RTE) [1,2]. Several numerical solution methods have been developed for radiative heat transfer problems and today most of researchers apparently use one of four methods: (a) the zonal method (ZM), (b) the Spherical Harmonics (P_N-Approximations) and its variations, (c) the Monte Carlo method (MCM), and (d) the discrete ordinates method (DOM) or its more modern form, the finite volume method (FVM). Another method is the discrete transfer method (DTM), which combines features of the DOM, ZM and MCM [3].

In particular, the DOM has been applied to, and optimized for, general radiative heat transfer problems, primarily through the pioneering works of Fiveland [4–7] and Truelove [8–10]. In the last two decades, the DOM, which is used in the present work, and the finite volume method have received great attention and have emerged as one of the most popular methods, providing a good

* Corresponding author.
E-mail addresses: jxaman@cenidet.edu.mx (J. Xamán), ivett@cicese.mx (I. Zavala-Guillén), ihernan@cenidet.edu.mx (I. Hernández-López), uriarte.fj@cenidet.edu.mx (J. Uriarte-Flores), ivan.hernandezp@ujat.mx (I. Hernández-Pérez), edgar.macias@ujat.mx (E.V. Macías-Melo), karla.aguilar@ujat.mx (K.M. Aguilar-Castro).

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Nomenclature

G	incident radiation, W m^{-2}
I	radiation intensity, W m^{-2} sr^{-1}
$I(\vec{r}, \Omega)$	radiation intensity in the position $\vec{r}$ and direction Ω, W m^{-2} sr^{-1}
$I(\vec{r}, \Omega')$	radiation intensity in the position $\vec{r}$ and incoming direction Ω; W m^{-2} sr^{-1}
$I(\vec{r}_w, \Omega)$	radiation intensity at the wall in the position $\vec{r}$ and direction Ω, W m^{-2} sr^{-1}
$I_b(\vec{r})$	blackbody radiation intensity in the position $\vec{r}$, W m^{-2} sr^{-1}
I_w	radiation intensity at the wall boundary condition, W m^{-2} sr^{-1}
k	constant of the *NWF* (intercept of the linear function)
mm	constant of the *NWF* (slope of the linear function)
M	number of discrete directions
$\hat{n}$	outward unit vector normal
$\vec{r}$	position vector
S	source term, W m^{-2}
w	quadrature weight
x, y	dimensional coordinates, m

Greek symbols

β	extinction coefficient, m^{-1}
γ	direction cosine
Δx	x-control volume thickness, m
Δy	y- control-volume thickness, m
ε_w	wall surface emissivity
κ_a	absorption coefficient, m^{-1}
μ,ξ,η	direction cosines in the x-, y-, z- directions
ρ_w	wall surface reflectivity
σ_s	scattering coefficient, m^{-1}
ϕ	general dependent variable
$\Phi(\Omega, \Omega')$	scattering-phase function
Ω	solid angle

Subscripts

b	Blackbody
C	central node
D	downstream node
e, w, n, s	east, west, north and south control volume faces
E, W, N, S	nodes neighbors of the node P
f	control volume face
N	north node
P	node of reference
S	south node
U	upstream node
W	west node

Superscripts

$'$	incident direction
$-$	indicate a normalized variable
m	discrete direction
n	current iteration
$n-1$	previous iteration

compromise between accuracy and computational economy. Today, they are probably the most popular RTE solvers [3].

The DOM is based on the numerical solution of the RTE for a set of discrete directions spanning the total solid angle range of 4π, replacing the integrals over direction (solid angle) by numerical quadratures.

Two major shortcomings of this method (DOM) that may strongly affect the solution accuracy are the ray effect and numerical scattering (false scattering), which were discussed in Refs. [11–13].

The ray effect is a consequence of angular discretization. It arises from the approximation of the continuous angular variation of the radiation intensity field by a discrete set of radiation intensities in specified ordinate directions. It is independent of the spatial discretization. Ray effects may be mitigated by refining the angular discretization or by using the modified discrete ordinates method [14].

The numerical scattering is associated with the spatial discretization scheme, and it is independent of the angular discretization. It arises in multidimensional problems when the radiation beams are not aligned with the grid lines. An evaluation of spatial discretization schemes used in the discrete ordinates method was presented in [15–17].

Although Raithby [18] describes that errors due to directional and spatial discretization tend to cancel. This cancellation should not be relied on; the spatial and directional discretizations should be made sufficiently fine that the error due to each is acceptable small, and any error cancellation that occurs should be welcomed as a bonus.

The DOM or the finite-volume methods require the evaluation of the radiation intensity at the cell faces of the control volumes that define the computational grid. Hence, the radiation intensity at the cell faces must be related to the radiation intensity at the grid nodes, which constitute the unknowns of the spatial discretized RTE. This relation was initially accomplished using either the STEP or the DIAMOND discretization schemes, which are the counterpart of the upwind and central difference scheme in Computational Fluid Dynamics (CFD). On the other hand, high-order resolution differencing schemes, initially developed by the CFD community, have also been used to solve the RTE using DOM. The SMART scheme [19] was used in [20], and the MINMOD [21], MUSCL [22], CLAM [23], and SMART schemes were used in [24]. Multiple high-order schemes were used by Coelho [25]. As in CFD, it has been shown that the radiation intensity field computed using these schemes is much more accurate than that obtained using the STEP scheme. The genuinely multidimensional (GM) discretization of the radiative transfer equation was proposed by Balsara [26]. This discretization minimizes the diffusion in the free streaming limit. Ismail and Salinas [27] used the genuinely multidimensional schemes to solve radiative heat transfer in a rectangular enclosure composed of diffusely emitting and reflecting boundaries and containing homogeneous media that absorbs, emits and scatters radiation.

Initially, in CFD some High-Order (HO) schemes (CENTRAL, QUICK) suffered from convergence difficulties and they had oscillatory behaviors. The main problem associated with the HO schemes was their boundedness. The difficulties associated with the development of reliable HO schemes stem from the conflicting requirements of accuracy, stability, and boundedness. The solutions predicted with HO schemes are more accurate than those calculated using the first order STEP scheme, but HO schemes tend to provoke oscillations.

To suppress oscillations, over the last three decades researchers eliminated this shortcoming of the HO schemes. The deficiency was removed through the introduction of the convection boundedness criterion [19,28], which led to the development of new families of High-Resolution (HR) schemes in the context of the Normalized Variable Formulations – NVF [29], the Normalized Variable and Space Formulation – NVSF [30], and the concept of Total Variation Diminishing (TVD), TVD schemes have been specially formulated to achieve oscillation-free solutions and they have proved to be useful in CFD calculations [21–23,31–34]. However, bound-