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Three-dimensional polarized radiative transfer simulation using discontinuous finite element method

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ABSTRACT

The polarized radiative transfer problem in three-dimensional scattering media is numerically solved by the discontinuous finite element method (DFEM). The discrete elements in the DFEM simulation are assumed to be discontinuous and the solution domain is connected by modeling the boundary numerical flux between adjacent elements, which makes the DFEM numerically stable for solving the radiative transfer equation. The shape functions are constructed on each element and the transformations between a general element and the standard element are presented. The accuracy of the DFEM for three-dimensional polarized radiative transfer is validated by comparing DFEM solutions with the published data in the literature. The DFEM is then applied to study the polarized radiative transfer problems in a cubic medium exposed to an external beam and in a cubic medium with an emitting surface. The distributions of the Stokes vector components are obtained and discussed.

1. Introduction

Polarized radiative transfer within a participating medium has attracted the interest of many researchers due to its wide applications in the fields of atmosphere and ocean systems [1–4], optical imaging [5], biomedical optics [6], and astronomy [7,8]. If the polarization nature of radiation is ignored, the polarized radiative transfer problem is reduced to the scalar radiative transfer problem where only the approximated intensity of the radiation is solved. Exact radiative transfer solutions preserve the polarization nature of the radiation, which is generally referred to vector radiative transfer [9–12]. However, due to the complexity of the vector radiative transfer equation (VRTE) in which four Stokes parameters needs to be solved, analytical solutions for the VRTE are a critical task. Researchers have been attempting to solve the VRTE and some numerical techniques [13–27] have been successfully applied to predict the radiative transfer process in participating media. Evans and Stephens [28] presented a summary of several commonly used polarized radiative transfer models and their applications. Kokhanovsky et al. [29] carried out an inter-comparison of several different numerical methods for the vector radiative transfer case of an underlying black surface. Benchmark results for vector radiative transfer in the cases of molecular, aerosol and cloudy multiple scattering atmosphere are given in [29]. Very recently, the International Polarized Radiative Transfer of the International Radiation Commission [30] initiated a model competition and the results for the one-dimensional plane-parallel model were summarized.

The grid-based discontinuous finite element method (DFEM), which is first proposed by Reed and Hill [31] and combines the salient figures of finite element method (FEM) and finite volume method (FVM) [32], is one of the most popular numerical methods for solving the integral-differential equations due to its numerical stability and flexibility in mesh grids. In recent years, the DFEM has attracted significant consideration and the previous work [33–35] shows the potential of DFEM for solving radiative transfer problems. Very recently, the DFEM has been extended for the first time to solve the radiative transfer problem considering the polarization effect [36–38], those studies show that the DFEM is accurate and stable for solving the VRTE in scattering media. However, the performance of the DFEM has only been examined in one- and two-dimensional cases. To the authors' best knowledge, there is no work reported in the literature dealing specifically with the application of DFEM for the polarized radiative transfer problems in three-dimensional (3D) media. In this paper, the DFEM formulation is applied to polarized radiative transfer problems in 3D scattering media and the Stokes vector component distributions are discussed.

The outline of this paper is as follows. In the following section, the mathematical formulation, DFEM discretization of the VRTE, and the construction of shape functions are presented. In Section 3, the accuracy of the proposed algorithm for 3D polarized radiative transfer is validated by comparing the DFEM results with those in

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Nomenclature

$\mathbf{I}$	Stokes vector $\mathbf{I}=(I, Q, U, V)^T$, W/(m^2·sr)
$\boldsymbol{\Omega}$	radiation direction
$\mathbf{S}$	source term function
$\boldsymbol{\beta}$	extinction matrix
$\mathbf{Z}$	scattering phase matrix
$\mathbf{R}$	reflection matrix
$\mathbf{n}$	unit normal vector
N_{elm}	number of discrete elements
N_e	number of nodes on a discrete element
M_θ	number of discrete zenith angles
M_φ	number of discrete azimuthal angles
M	total number of discrete directions
I_{b}	black body emission, W/(m^2·sr)
β	extinction coefficient, m^{-1}
κ_{a}, κ_{s}	absorption and scattering coefficient, m^{-1}
ω	$\kappa_{\mathrm{s}}/\beta$, scattering albedo
q	irradiance, W/m^2
G	scalar irradiance, W/m^2
$\mathbf{r}$	general coordinate vector, m
x, y, z	general coordinates, m
$\mathbf{r}_{\mathrm{st}}$	normalized coordinate vector
ξ, η, γ	normalized coordinates
L_x, L_y, L_z	geometrical length of the domain, m
θ	zenith angle, rad
φ	azimuth angle, rad
$\boldsymbol{\Theta}$	scattering angle, rad
μ	$\mu=\cos\theta$, direction cosine
w	weight function
ϕ	shape function on general element
ψ	shape function on standard element

Subscripts

i, j	index for nodes

Superscripts

m, m'	index for directions

literature. In Section 4, the DFEM is applied to solve the VRTE in a cubic medium exposed to an external beam and in a cubic medium with a hot surface. Finally, some notable conclusions are drawn in the last section.

2. Theory

2.1. Governing equation

The discrete-ordinate form of the vector radiative transfer equation for an absorbing and scattering medium can be written as [1]

$$\boldsymbol{\Omega}^m \cdot \nabla \mathbf{I}(\mathbf{r}, \boldsymbol{\Omega}^m) + \beta \mathbf{I}(\mathbf{r}, \boldsymbol{\Omega}^m) = \mathbf{S}(\mathbf{r}, \boldsymbol{\Omega}^m), \tag{1}$$

where $\mathbf{I}=(I, Q, U, V)^{\mathrm{T}}$ is the Stokes vector with the superscript 'T' denoting the matrix transposition. I is the total-spectral radiation intensity, Q is the linear polarization aligned parallel or perpendicular to the z-axis, U is the linear polarization aligned ±45° to the z-axis and V is the circular polarization. $\boldsymbol{\Omega}$ is the radiation direction, $\boldsymbol{\beta}$ is the extinction coefficient matrix, and $\mathbf{S}$ is the source term written as

$$\mathbf{S}(\mathbf{r}, \boldsymbol{\Omega}^m) = \kappa_{\mathrm{a}} \mathbf{I}_{\mathrm{b}}(\mathbf{r}) + \sum_{m'=1}^{M} \mathbf{Z}(\mathbf{r}, \boldsymbol{\Omega}^{m'} \to \boldsymbol{\Omega}^m) \mathbf{I}(\mathbf{r}, \boldsymbol{\Omega}^{m'}) w^{m'}, \tag{2}$$

where $\mathbf{I}_{\mathrm{b}}=(I_{\mathrm{b}}, 0, 0, 0)^{\mathrm{T}}$ is the medium emission vector with I_{b} denoting the blackbody intensity, $\mathbf{Z}$ is the scattering phase matrix,

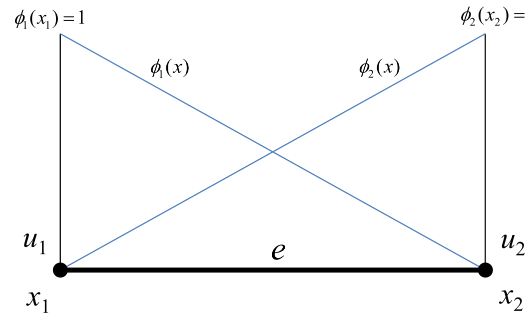

Fig. 1. One-dimensional general element and shape functions.

M is the total number of the discrete directions, $\boldsymbol{\Omega}^m$ and $\boldsymbol{\Omega}^{m'}$ are the discrete directions, $w^{m'}$ is the corresponding weight in direction $\boldsymbol{\Omega}^{m'}$ for angular quadrature [39].

In the case of the emission and reflection, the boundary condition is given as

$$\mathbf{I}(\mathbf{r}_{\mathrm{w}}, \boldsymbol{\Omega}^m) = \kappa_{\mathrm{a}} \mathbf{I}_{\mathrm{b}}(\mathbf{r}_{\mathrm{w}}) + \mathbf{R}_{\mathrm{s}} \mathbf{I}(\mathbf{r}_{\mathrm{w}}, \boldsymbol{\Omega}^{m''}) + \frac{1}{\pi} \int_{\mathbf{n}_{\mathrm{w}} \cdot \boldsymbol{\Omega}^{m'} > 0} \mathbf{R}_{\mathrm{d}} \mathbf{I}(\mathbf{r}_{\mathrm{w}}, \boldsymbol{\Omega}^{m'}) |\mathbf{n}_{\mathrm{w}} \cdot \boldsymbol{\Omega}^{m'}| d\boldsymbol{\Omega}^{m'}, \tag{3}$$

where the subscript 'w' denotes variable values on boundary nodes, $\mathbf{n}_{\mathrm{w}}$ denotes the unit outward normal vector of the global boundary, $\boldsymbol{\Omega}^{m'}$ denotes the corresponding incident directions of the current diffusely reflected direction $\boldsymbol{\Omega}^m$, it meets the condition of $\mathbf{n}_{\mathrm{w}} \cdot \boldsymbol{\Omega}^{m'} > 0$, $\boldsymbol{\Omega}^{m''}$ denotes the corresponding incident direction of the current specularly reflected direction $\boldsymbol{\Omega}^m$, $\mathbf{R}_{\mathrm{s}}$ and $\mathbf{R}_{\mathrm{d}}$ are the specular and diffuse reflection matrices.

2.2. Shape function

In the DFEM application, the continuous computational domain is divided into a tessellation of small, non-overlapping, and interconnected sub-regions referred to as elements. Piece-wise approximations to the governing equation are given over these elements and the integral-differential equation is break down into a series of linear simultaneous equations. Thus, the space discretization (i.e., dividing the domain into discrete elements) procedure reduces the continuum problem, which has an infinite number of unknowns, to one with a finite number of unknowns at specified points referred to as nodes. By dividing the computational domain into a finite number of elements, and approximating the solutions for the VRTE in a piece-wise manner over these elements by a suitable known function, the solutions of the differential equation within the computational domain can be obtained. The functions employed on each element to represent solutions on any location within the element are called shape functions [40,41], and they are obtained as follows.

Fig. 1 shows a general 1D element e ($x \in [x_1, x_2]$) with two nodes defined on the Cartesian coordinate. By the Lagrangian interpolation theory, the unknown function u (temperature and radiation intensity for example) is approximated by a function $u(x)$ local to the element

$$u(x) = \alpha_1 + \alpha_2 x, \tag{4}$$

where the parameters α_1 and α_2 are constants to be determined. Since there are two constants in Eq. (4), the information on the two adjacent nodes is required to determine the values of α_1 and α_2, namely

$$u_1 = \alpha_1 + \alpha_2 x_1, \tag{5a}$$

$$u_2 = \alpha_1 + \alpha_2 x_2. \tag{5b}$$