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Scattering and attenuation of electromagnetic waves by partly charged particles

Jùn Zhou [a,b], X. Dou [a,b], Li Xie [a,b,*]

[a] Key Laboratory of Mechanics on Environment and Disaster in Western China, Ministry of Education, 730000, China
[b] College of Civil Engineering and Mechanics, Lanzhou University, Lanzhou 730000, Gansu, China

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ABSTRACT

The scattering of electromagnetic waves (EMWs) by partly charged particles is investigated by Mie theory. For particles much smaller than the wavelength of EMW, the scattering properties are significantly affected by the net surface charges but are not affected by the location of charged area, and the extinction cross section and scattering cross section, as well as the signal attenuation due to charged particles, monotonously increase with the size of charged area and reach maximum when the particle is overall charged, as the surface charge densities is within several hundred $\mu C/m^2$. Moreover, given the distribution of particle sizes a closer agreement between the theory and measurement of signal attenuation due to charged sand/dust storms can be achieved by taking the charges into consideration and assuming all particles have the same size of charged area and surface charge density in theory calculations.

1. Introduction

At sand/dust storm regions, remote sensing is used for the observation of sand/dust storms, to observe the visibility, the density of particles, and etc. Generally, remote sensing uses the scattering properties of electromagnetic waves (EMWs), including the extinction/absorption, scattering and attenuation of electromagnetic waves caused by sand/dust storms. On the other hand, the operation of telecommunication at sand/dust storm regions could be worsened by sand/dust storms. Therefore, to ensure the accuracy of remote sensing and reliability of telecommunications in sand/dust storms, the scattering properties of EMWs need being investigated sufficiently. Much research has been conducted on the scattering and attenuation of EMWs caused by sand/dust storms or sand/dust particles [1–4]. And the calculated results had large errors as compared with experiment results [2–4]. In these work, the particles were assumed neutral, i.e., the particles did not carry net surface charges. However, it has been well known that the particles in sand/dust storms generally carry large amount of charges and can generate strong electric field [5]. Sand/dust particles are charged due to many reasons, such as the friction [6], contact [7,8], and frequent collisions between particles [9,10]. There were several reasons to explain why theoretical results of scattering and attenuation had large errors [2–4]. And it was pointed out that the net surface charges carried by sand/dust particles should have significant contribution to the scattering properties and attenuation [11].

Recently, the effect of surface charges on the scattering of EMWs by charged particles was investigated [12–18] based on Mie theory, and it was found that the scattering properties could be significantly affected as compared with those of uncharged particles [12–19]. In these researches the particles were assumed be overall charged uniformly on surfaces. However, according to existing researches on the electrification of particles, charge carriers generally transfer between two insulator's surfaces brought into contact [6–10], which means a particle very probably carries net charges only on a part of its surface. Based on Rayleigh's scattering theory, it was shown that the extinction of EMWs by a particle was enhanced when it was charged and the enhancement was maximal when a hemisphere of the particle was charged [20], while the enhancement vanished when the particle was overall charged uniformly on its surface [20]. The reason for this to happen was that the contribution of net surface charges to the scattering of EMWs was the static electric dipole induced by the net surface charges. The strength of the static electric dipole was maximal when a hemisphere was charged and always vanished due to symmetry when the particle was overall charged uniformly. Hence, whatever the density of surface charges was, the effect of net surface charges on the scattering of EMWs always vanished when the particle was overall charged uniformly. However, compared with the case that a hemisphere was charged, an overall charged particle would have more charges that could be excited by EMW sand radiate/absorb

* Corresponding author at: Key Laboratory of Mechanics on Environment and Disaster in Western China, 730000, China.
E-mail address: xieli@lzu.edu.cn (L. Xie).

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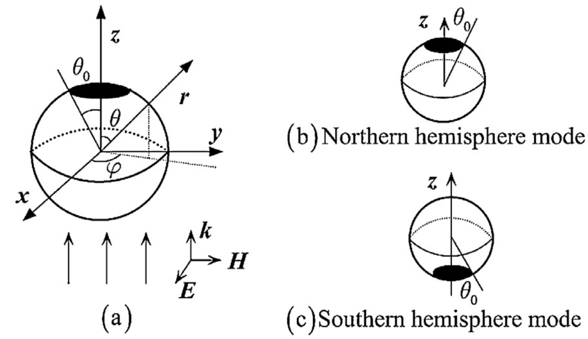

Fig. 1. (a) Schematic of a partly charged particle illuminated by an x-polarized electromagnetic wave propagating along z-axis, i.e., the wave vector **k** is in z-direction. **E** and **H** denote the electric field and magnetic field vectors, respectively. (b) Northern hemisphere mode, namely, the particle is charged at the crown (black), whose centroid is the North pole of the particle. (c) Southern hemisphere mode, in which the centroid of charged crown (black) is the South Pole. Here, the angle θ_0 measures the size of charged area.

energy, why should the effect of surface charges always vanish? A research on the scattering of partly charged particles was conducted in [21] based on Mie theory, however, the solutions of Mie scattering coefficients given by [21] were not correct as pointed out by [17].

For the operation of telecommunication and remote sensing at sand/dust storm regions, it is important to find out how signals are affected by charged sand/dust storms, e.g., what the error of theoretical results could be affected by the net charges carried by particles, the lower and upper bound of the signal attenuation under the effect of net charges, and etc. Motivated by this, the effect of surface charges on the scattering of EMWs by partly charged particles, including extinction cross section, scattering cross section and signal attenuation, are investigated based on Mie theory in this paper.

The paper is organized as follows: in Section 2, the scattering problem of EMWs by partly charged particles is solved, and linear equations for solving Mie's scattering coefficients are established; the effect of surface charges on the extinction, scattering cross section and signal attenuation by partly charged particles is analyzed and discussed in Section 3; and conclusions are given in Section 4.

2. Scattering field of a partly charged particle

As shown in Fig. 1(a), a charged spherical particle with radius a is illuminated by an x-polarized planar harmonic EMW propagating along z-axis. The electric field of the incident EMW is of the form $\mathbf{E}_{in}=E_0e^{-i\omega t}\mathbf{e}_x$, in which E_0 denotes the amplitude of electric field and $\omega=2\pi f$, where f denotes the frequency of the EMW. Here, $\mathbf{e}_x$ is the unit vector of x-axis. The unit vector $\mathbf{e}_x$ can be written in spherical coordinate as $\mathbf{e}_x=\sin\theta\cos\varphi\mathbf{e}_r+\cos\theta\cos\varphi\mathbf{e}_\theta-\sin\varphi\mathbf{e}_\varphi$. The particle is assumed be charged at a crown-shaped area, as shown in Fig. 1(b) and (c), where θ_0 measures the size of the area. For the case of Fig. 1(b), the charged area locates at the northern hemisphere (i.e. for $z>0$) of the particle, so we call it a Northern Hemisphere Mode (NHM), while the case in Fig. 1(c) is called Southern Hemisphere Mode (SHM). The particle medium and the ambient medium are assumed isotropic and homogeneous. Let $\mathbf{E}_1$ and $\mathbf{H}_1$ denote the fields inside the particle, $\mathbf{E}_2$ and $\mathbf{H}_2$ denote the fields of the ambient. The electric field $\mathbf{E}_i$ and magnetic field $\mathbf{H}_i$ ($i=1, 2$) satisfy the following wave propagation equation [13],

$$\begin{cases}\nabla^2\mathbf{E}_2+k_2^2\mathbf{E}_2=0, & \nabla^2\mathbf{H}_2+k_2^2\mathbf{E}_2=0\\ \nabla^2\mathbf{E}_1+k_1^2\mathbf{E}_1=0, & \nabla^2\mathbf{H}_1+k_1^2\mathbf{E}_1=0\end{cases} \tag{1}$$

where k_1 and k_2 are the wave numbers inside and outside the particle, respectively. $k_i^2=\omega^2\mu_i\varepsilon_i$ in which μ_i and ε_i ($i=1, 2$) denote the permeability and permittivity of the medium, respectively. We consider the medium outside of the particle as vacuum, i.e. $\mu_2=\mu_0$ and $\varepsilon_2=\varepsilon_0$. Here, $\mu_0=1.2566\times10^{-6}$ H/m (SI unit) and $\varepsilon_0=8.8542\times10^{-12}$ F/m (SI unit), are the vacuum permeability and the vacuum permittivity. $\mu_1=\mu_0\mu_r$ and $\varepsilon_1=\varepsilon_0\varepsilon_r$, where μ_r and ε_r are the relative permeability and the relative permittivity of the particle medium. Here, the magnetism of the particle is generally weak, i.e., $\mu_r=1$. Following Mie theory, the field outside the particle is the superposition of the incident field and the scattering field, i.e. $\mathbf{E}_2=\mathbf{E}_{in}+\mathbf{E}_s$, and $\mathbf{H}_2=\mathbf{H}_{in}+\mathbf{H}_s$, where $\mathbf{E}_s$ and $\mathbf{H}_s$ denote the scattering fields of the particle. Assume that there exists surface current **K** on the particle [13]. Hence, the boundary condition has the following form [13],

$$\begin{cases}(\mathbf{E}_2-\mathbf{E}_1)\times\mathbf{n}=0\\ (\mathbf{H}_2-\mathbf{H}_1)\times\mathbf{n}=\mathbf{K}\end{cases} \tag{2}$$

in which **n** stands for the normal of particle's boundary. The surface current **K** is calculated by Ohm's law, i.e. $\mathbf{K}=\sigma_s\mathbf{E}_{1,\tau}$, where $\mathbf{E}_{1,\tau}$ denotes the tangential component of the electric field on the surface. σ_s is called surface conductivity of the particle, a phenomenological parameter to describe the surface current due to the motion of net surface charges driven by electric field [12,13]. Under given surface conditions, including the density of surface charges η_s and temperature T, the surface conductivity can be calculated by Drude model [16–18] $\sigma_s=\frac{\eta_s q/M}{i\omega+\gamma_s}$, where η_s is the density of surface charge and q/M the charge-to-mass of charge carriers, e.g. $q/M=1.7587\times10^{12}$ C/kg for electrons. The parameter γ_s is calculated by $\gamma_s=k_BT/\hbar$, where k_B denotes the Boltzmann constant and $\hbar$ is Planck's constant [16–18].

The scattering problem, i.e., the wave propagation Eq. (1) plus the boundary condition (2), can be solved by Mie theory, in which the incident field $\mathbf{E}_{in}$ is expanded in vector spherical harmonics (3).

$$\mathbf{E}_{in}=E_0\sum_{n=1}^{\infty}i^n\frac{2n+1}{n(n+1)}\left[\mathbf{M}_{omn}^{(1)}-i\mathbf{N}_{emn}^{(1)}\right],\quad m=1. \tag{3}$$

Here, $i=\sqrt{-1}$. $\mathbf{M}_{omn}^{(j)}$ and $\mathbf{N}_{emn}^{(j)}$ are vector spherical harmonics defined by (4) and (5).

$$\mathbf{M}_{omn}^{(j)}=z_n^{(j)}(\rho)[\cos m\varphi\pi_n(\cos\theta)\mathbf{e}_\theta-\sin m\varphi\tau_n(\cos\theta)\mathbf{e}_\theta] \tag{4}$$

$$\mathbf{N}_{emn}^{(j)}=n(n+1)\sin\theta\pi_n(\cos\theta)\frac{z_n^{(j)}(\rho)}{\rho}\mathbf{e}_r+\frac{\left[z_n^{(j)}(\rho)\right]'}{\rho}\times[\cos m\varphi\tau_n(\cos\theta)\mathbf{e}_\theta-\sin m\varphi\pi_n(\cos\theta)\mathbf{e}_\varphi] \tag{5}$$

where $\rho=kr$ and k denotes the wave number. r, θ and φ are spherical coordinates and $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\varphi$ are the corresponding unit vectors. $\pi_n(\cos\theta)=\frac{P_n^1(\cos\theta)}{\sin\theta}$ and $\tau_n(\cos\theta)=\frac{d}{d\theta}\frac{P_n^1(\cos\theta)}{\sin\theta}$, where $P_n{}^1(\cdot)$ is the associate Legendre polynomial. For $j=1$ and 3, $z_n^{(j)}(\rho)$ is the first and third kind of spherical Bessel function [13].

Here, the charges are supposed uniformly distributed on the charged crown. That is, the charge density η_s is constant on the crown. According to the theories of surface conductivity [12–18], the surface conductivity σ_s is a constant under given surface conditions. Especially, σ_s is proportional to the density of surface charges, which means $\sigma_s=0$ for uncharged surfaces ($\eta_s=0$). Therefore, for a partly charged particle of NHM shown in Fig. 1(b), the surface conductivity $\sigma_s(\theta)$ is a constant σ_s for $0\le\theta\le\theta_0$ and $\sigma_s(\theta)=0$ otherwise; for the case of SHM in Fig. 1(c), $\sigma_s(\theta)$ is a constant σ_s for π-$\theta_0\le\theta\le\pi$ and $\sigma_s(\theta)=0$ otherwise. That means the surface conductivity σ_s is independent from the spherical coordinate φ, i.e., σ_s is constant for all φ in [0, 2π] for both NHM