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# On numerical orientation averaging with spherical Fibonacci point sets and compressive scheme

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ABSTRACT

For light scattering computations, we propose to perform orientation averaging with spherical Fibonacci point sets. Furthermore, a compressive scheme is introduced to recover light scattering quantities for all orientations based on the compressive sensing theory that exploits the priori information that these scattering quantities are sparse in the spherical harmonic domain. The scheme solves convex optimization problem by minimizing the $l_1$-norm of spherical harmonic expansion coefficients of light scattering quantities. Combining with spherical Fibonacci point sets, this compressive scheme can achieve highly accurate recovery results with fewer orientations than conventional orientation averaging schemes. It is indicated that the proposed compressive orientation averaging scheme with Fibonacci point sets is straightforward to implement and shows good performance in error convergence.



## 1. Introduction

Particle orientation is of significant importance for investigating optical or aerodynamic properties of small particles. In numerical light scattering simulations [12], it is often assumed that particle orientations are uniformly randomly distributed. But in some meteorology studies [3], nonuniform distributions of atmospheric particle orientations are also used with the assumption that particles are horizontally oriented. For optical properties of small particles [11], we can simulate light scattering with various computational methods [6,12,17] and integrate optical quantities over the particle orientation domain. For example, random orientation averaging can be performed analytically with T-matrix method. For other fixed orientation methods like FDTD [17] or DDA [6], various numerical integration schemes are needed to obtain mean properties.

In FDTD or DDA, the CPU time is linearly proportional to the number of orientations, since we obtain one fixed orientation result in each simulation. Hence, an efficient orientation averaging scheme would be beneficial to reduce the simulation time consumption. Based on this, Okada [14] studied a numerical orientation averaging with Quasi-Monte Carlo (QMC) sampling point sets, which reduces the variance compared with classical Monte Carlo sampling point sets. Penttilä et al. [15] proposed another orientation averaging scheme with Lebedev-Laikov cubature, which can be defined as point sets on the sphere with octahedral rotation symmetry. Recently, several studies [7,9,16] have shown that spherical Fibonacci lattices are quite appropriate for numerical integration on spherical domain. Here, we would like to introduce the use of spherical Fibonacci point sets for orientation averaging in light scattering computations.

One of the most time-consuming step for light scattering simulations is conducting the orientation averaging of the scatterer. All these numerical integration schemes above, however, have not taken the symmetry of objects into account. To take symmetry into consideration, we need to turn to spherical harmonics, of which scattering quantities on the unit sphere can be written as linear combinations. It would also be advantageous to evaluate the performance of the corresponding quadrature scheme by determining the ratio of the number of spherical harmonics expansions to the number of integration points. Given the quadrature scheme and points, it would be interesting to know how many terms of spherical harmonics expansions we can recover. This question is answered by the sampling theorem on the sphere [10], which states that how exactly a band-limited spherical function can be recovered from samples. Furthermore, with the arise of the compressive sensing (CS) theory [5], it is possible to recover a sparse band-limited function from much fewer points than conventional sampling theorems. The second and major contribution of this paper is introducing a compressive orientation averaging scheme with the use of compressive sensing theory to recover the spherical distributions of quantities in light scattering computations.

The remaining of this paper is outlined as follows. In section 2, we explain the relation between particle orientation as well as the direction statistics. In section 3, we introduce two sampling meth-

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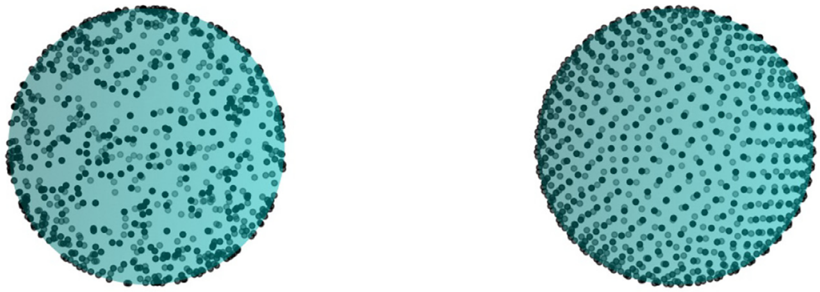
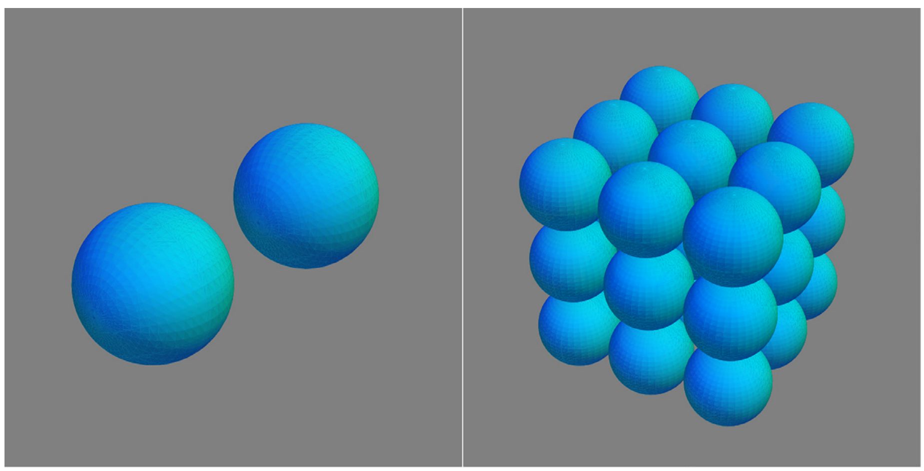


(a) Monte Carlo point set. (b) Fibonacci point set.

**Fig. 1.** Examples of two point sets of size 800 generated by the algorithms described above. Left: Uniform sampling points generated by Monte Carlo method. Right: Fibonacci lattice on the unit sphere.



(a) Bi-sphere (b) Multi-spheres

**Fig. 2.** Scatterers used in the light scattering computations. Left: bi-sphere cluster. Right: multi-sphere cluster.

ods on the unit sphere, which are Monte Carlo sampling and Fibonacci lattice sampling, and propose a novel scheme to recover a function based on compressive sensing. Then, we formulate a compressive orientation averaging scheme for light scattering computations. In section 4, we verify the proposed scheme in the simulations for light scattering by multi-sphere clusters. Finally, we make the conclusion in section 5.

## 2. Orientation averaging and direction statistics

Recently, Mishchenko and Yurkin [13] provided a rigorous mathematical explanation for particle orientations in light scattering computations. Based on their work, we try to link particle orientations to direction statistics. In the following, we will show that direction statistics provide a rigorous statistical framework, with which (uniformly) random orientation and (non-uniformly) random orientation can be unified. It is known that particle orientations are uniquely parameterized by three Euler angles $(\alpha, \beta, \gamma)$. Accordingly, orientation-averaged quantities such as scattering phase matrix can be interpreted as expected values over Euler angles. Instead of rotating the object, it is equivalent to rotate the propagating direction of the incident wave. In methods such as FDTD, the simulation of each incident direction is decomposed into vertically and horizontally polarized components, in which the computed scattering properties of that incident direction are taken as the mean of these two situations. The three Euler angles parameters are reduced to two. Therefore, a probability density function (PDF) for incident directions can then be expressed by the nonnegative spherical function $f(\theta, \phi)$ with the normalization condition:

$$\int_0^{\pi} \int_0^{2\pi} f(\theta, \phi) \sin\theta d\theta d\phi = 1 \tag{1}$$

where $f(\theta, \phi) = \frac{1}{4\pi}$ for uniform random distribution. In our simulations, we assume all the model particles to be uniformly randomly oriented. Given a set of $N$ incident directions, orientation averaging schemes become spherical integrations. And the orientation averaging value can be written as

$$\mathbb{E}[q] = \int \int q(\theta, \phi) f(\theta, \phi) \sin\theta d\theta d\phi \tag{2}$$

$$\approx \frac{1}{N} \sum_{j=1}^{N} w_j q(\theta_j, \phi_j) \tag{3}$$

where $\{w_j\}$ denotes the weights and $\mathbb{E}[*]$ denotes the expected value of a spherical function $q(\theta, \phi)$. For the uniform distribution and uniformly sampled points, we have $w_j = 1$.

## 3. Orientation integration and compressive sensing

### 3.1. Orientation integration with Fibonacci point sets

If particle orientations are uniformly distributed, it is straightforward to apply Monte Carlo method to sample points uniformly