Journal of Quantitative Spectroscopy & Radiative Transfer 206 (2018) 105–116

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Effective scaling factors in non-uniform gas radiation modeling

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ARTICLE INFO

Article history:
Received 7 July 2017
Revised 22 September 2017
Accepted 27 October 2017
Available online 31 October 2017

Keywords:
Gas radiation
Non-uniform
Effective scaling factor
ℓ-distribution
Scaled models
Scaled-k

ABSTRACT

This paper is devoted to non-uniform approximations for the radiative properties of gases based on effective scaling factors. These methods are recognized as more accurate than other established techniques such as the Curtis-Godson approximation in statistical narrow band modeling, or the Correlated-k assumption. An analytical solution is proposed to calculate these scaling factors and a comprehensive description of the method to derive its parameters from high resolution spectra is given. Practical implications of the results of the present work are quite large, as non-uniform techniques based on effective scaling factors can be applied to any model form. The main value of this work is to render possible and computationally realistic the use of this category of non-uniform approximations for radiative heat transfer calculations.

1. Introduction

For many years, the scaling approximation, which consists in assuming that the ratio between two spectra in distinct thermophysical states is a constant with respect to the wavenumbers, was the only possible way to handle radiative heat transfer problems in non-uniform gaseous media. This simple idea, which is known to provide inaccurate results in a general frame as gas spectra in distinct states are not linearly dependent, is however the building block of many non-uniform treatments proposed all along the past century. Various techniques are in fact derived from the scaled view such as:

1/ non-uniform approximations involving two instead of one scaling coefficient, like in the Curtis-Godson (CG) approximation [1],2/ the so-called Correlated-k technique [2], which mostly consists of the definition of scaling coefficients that depend on the values of the absorption coefficient in one particular state chosen as a reference.

3/ the Godson-Weinreb-Neuendorffer GWN [1,3] / Emissivity Growth Approximation EGA [4] or Scaled-k [5] approaches, which do not try to provide an explicit way to evaluate effective scaling factors but instead rely on an implicit definition of these quantities (see Eq. (13) later in this paper).

These last methods are widely recognized as more accurate than those based on explicit definitions of finite numbers of scaling coefficients: see Ref. [1], chapter 12, in which comparisons between the GWN and CG methods are described; or Ref. [5] where it is shown that the Scaled-k approach outperforms the Correlated-k method for radiative heat transfer in highly non-uniform situations. Despite this advantage of non-uniform techniques based on effective scaling factors, it must be recognized that this category of methods has not embraced the same interest as other, in fact usually simpler, non-uniform approximations. This is mainly because, in general, solving the implicit equation Eq. (13) to determine the effective scaling factor involves iterative numerical techniques that increase significantly the computational cost of the approach compared to explicit methods.

Recently, the ℓ-distribution approach was proposed [6]. This approximate model for the radiative properties of gases is founded on a formalism that: 1/ provides accurate approximations in uniform media and 2/ allows solving very efficiently the implicit equation involved in the definition of effective scaling factors. While developing this method, the author of the present paper noticed that the literature on effective scaling approximation is quite meager. Indeed, the non-uniform approximation (viz. the implicit equation) is always introduced in an intuitive way without specification of the assumptions that may lead to this particular equation. Accordingly, the existing literature on effective scaling factors does not permit studying in depth some of their properties (the functional form of the solution, for instance) nor it can be helpful to improve the technique further.

The aim of the present paper is to provide insights into the assumptions made to extend the usual concept of constant scaling coefficients to effective ones. Starting from a truly scaled situation, we first explain how the concept of constant scaling coefficient can be extended to real spectra. The main assumption required is shown to be the statistical independence between a spectrum, cho-

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https://doi.org/10.1016/j.jqsrt.2017.10.019
0022-4073/

Nomenclature

EN	radiative energy emitted in a given spectral region
F	distribution function of spectral scaling coefficients – Eq. (7)
g	cumulative k-distribution – Eq. (5)
ℓ	inverse of the transmission function (cm) as defined in Section 2
L	gas path length (cm)
M	joint distribution function of spectral and absorption coefficients – Eq. (10)
$u(L)$	effective (path dependent) scaling factor
$u(L)\cdot L$	effective scaling function (cm) – Eq. (14)
u_η	spectral scaling coefficient
U	constant scaling factor
x	abscissas of a Gauss-Legendre quadrature over [0,1]

Greek symbols

$\Delta\eta(k)$	set of wavenumbers $\{\eta \in \Delta\eta$ such that $k < \kappa_\eta^1 < k + dk\}$
$\Delta\eta(k)$	width of the set of wavenumbers $\{\eta \in \Delta\eta$ such that $k < \kappa_\eta^1 < k + dk\}$
κ_η	spectral absorption coefficient (cm^{-1})
η	wavenumber (cm^{-1})
μ_S	$= [\frac{1}{\Delta\eta}\cdot\int_{\Delta\eta}(\kappa_\eta)^s d\eta]^{1/s}$ generalized mean absorption coefficient with real exponent s
ρ_{SP}	Spearman's rank correlation coefficient – Eq. (11)
τ	transmission function; transmissivity
ω	weights of a Gauss-Legendre quadrature over [0,1]
ξ	dummy variable inside [0,1]

Subscripts

12	related to the non-uniform path $L = L_1 + L_2$
b	blackbody
P	Planck mean
R	Rosseland mean
S	associated with the set of wavenumbers defined as $\{\eta \in \Delta\eta$ such that $\mu_S < \kappa_\eta^1 < \mu_S + d\mu_S\}$

Superscripts

eff	effective
GG	gray gas
1,2	state 1 or 2 of the gas
$\Delta\eta$	width of the spectral interval for the averaging of spectral properties

Other notations

$f \circ g$	represents the functional composition of f and g i.e. $f \circ g(x) = f[g(x)]$
$\tau_{ij}^{\Delta\eta}(L_p, L_q)$	$= \frac{1}{\Delta\eta}\int_{\Delta\eta}\exp(-\kappa_\eta^i L_p - \kappa_\eta^j L_q)d\eta$ *Ex.* $\tau_{11}^{\Delta\eta}(L_1, L_2) = \frac{1}{\Delta\eta}\int_{\Delta\eta}\exp(-\kappa_\eta^1 L_1 - \kappa_\eta^1 L_2)d\eta$

Abbreviations

EGA	Emissivity Growth Approximation - Ref. [4]
GWN	Godson-Weinreb-Neuendorffer's method - Refs. [1,3]
LBL	Line-By-Line
MoD	Measure of Dependence – Refs. [12,13]
SNB	Statistical Narrow Band model – Ref. [1]

sen as a reference, and the spectral scaling coefficients defined as the ratio between any other spectrum and this reference. Based on this assumption of statistical independence, an explicit mathematical formula – Eq. (27) -, which is the core finding of the present work, is derived for effective scaling functions. The full method to construct this function for radiative heat transfer applications is also described and assessed against reference LBL calculations in non-uniform situations.

The main practical implication of the results provided in the present work is that effective scaling functions require the specification of parameters that are hard to define without optimization. To some extent, one faces the same problem as encountered in Correlated-k models in which the existence of a strictly increasing function that associates spectra in distinct states is assumed. This function cannot be identified directly from LBL data because it is founded on assumptions about the statistical properties of gas spectra, not on a true description of high resolution data. Accordingly, the only way to define this function to associate spectra in distinct states is implicit, by using a relationship that involves the cumulative distributions of the absorption coefficients in the various thermophysical states. In the same way, within the frame of scaled models with variable scaling factors, the most relevant approach to derive effective scaling factors is to solve the implicit equation directly. This makes the ℓ-distribution approach undoubtedly the most efficient and accurate method for this purpose. This statement is discussed further in the paper. The paper also provides an explicit formulation to solve this equation, which can be used with any model form.

The main value of the present paper is to provide, to the best of the author's knowledge, the first detailed analysis of models based on effective scaling factors, from the derivation of the scaling function up to the analysis of the conditions required between spectra for this formulation to be acceptable. Results from the present work have strong implications for future developments of the ℓ-distribution approach but are not restricted to this approximate model: scaled-k methods, such as FSSK [5] or the recently proposed Scaled-SLW modeling [7], can benefit directly from results described here.

The paper is organized as follows. In the second section, the concept of effective scaling models is introduced. A simple case is treated in order to illustrate some characteristics of this kind of approaches. This section ends with a detailed derivation of effective scaling functions within the frame of narrow band models. In the third section, the full method to evaluate the coefficients that appear in the effective scaling function is described. Detailed statistical analysis of spectra together with their correlation with spectral scaling coefficients are given. Comparisons of the approximate model with LBL calculations show the relevancy of the method proposed for radiative heat transfer calculations.

All derivations provided in this work are restricted to the two-cell problem. This analysis is sufficient for application together with all existing methods based on effective scaling factors which either: 1/ assume the existence of some reference state and then scale any other state to this reference [5,7]; or, 2/ propagate the information about scaling factors along a non-uniform path in a step by step manner by coupling adjacent layers [3,4,6,11]. In both cases, only two distinct states of the gas are involved.

2. Model of effective scaling functions

2.1. Introduction to the concept of effective scaling factors / functions

Let us start by reminding some results related to scaled spectra. For this purpose, we consider a non-uniform layer discretized in two homogeneous isothermal sub-paths: the first one has a length L_1 and the gas is in the thermophysical state ϕ_1, the second path has a length L_2 and the state of the gas is ϕ_2. The spectral absorption coefficients in the two layers are κ_η^1 and κ_η^2 respectively. We will restrict here our analysis to narrow bands $\Delta\eta$ over which: 1/ the Planck function is constant; 2/ absorption coefficients κ_η^1 and κ_η^2 are strictly positive.

In the case of truly scaled spectra, the ratio $\kappa_\eta^2/\kappa_\eta^1 = U$ is a constant with respect to the wavenumbers. The transmissivity of the non-uniform path $L = L_1 + L_2$ averaged over the narrow band $\Delta\eta$,