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The Scaled SLW model of gas radiation in non-uniform media based on Planck-weighted moments of gas absorption cross-section

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ABSTRACT

The Scaled SLW model for prediction of radiation transfer in non-uniform gaseous media is presented. The paper considers a new approach for construction of a Scaled SLW model. In order to maintain the SLW method as a simple and computationally efficient engineering method special attention is paid to explicit non-iterative methods of calculation of the scaling coefficient. The moments of gas absorption cross-section weighted by the Planck blackbody emissive power (in particular, the first moment – Planck mean, and first inverse moment – Rosseland mean) are used as the total characteristics of the absorption spectrum to be preserved by scaling. Generalized SLW modelling using these moments including both discrete gray gases and the continuous formulation is presented. Application of line-by-line look-up table for corresponding ALBDF and inverse ALBDF distribution functions (such that no solution of implicit equations is needed) ensures that the method is flexible and efficient. Predictions for radiative transfer using the Scaled SLW model are compared to line-by-line benchmark solutions, and predictions using the Rank Correlated SLW model and SLW Reference Approach. Conclusions and recommendations regarding application of the Scaled SLW model are made.

1. Introduction

Among the global methods of gas radiation, the Spectral-Line-Weighted-sum-of-gray-gases (SLW) model has been demonstrated to be a simple, accurate, and efficient method of spectral modelling of radiation transfer in gaseous media [1]. A primary challenge of global methods is spectral modelling of radiation transfer in non-uniform media (non-isothermal and/or non-homogeneous scenarios), where the gas absorption spectrum changes with spatial location due to changes in local thermodynamic state. Approaches based on the assumption of correlated gas absorption spectra have been developed [1–6]. Greater accuracy in these approaches comes generally at the expense of greater sophistication, more complexity, greater difficulty in implementation, and higher associated computational cost. This work concerns itself with the scaled spectrum assumption in the context of the Spectral Line Weighted-sum-of-gray-gases (SLW) model.

The SLW method considered in this work is an extension and further development of the global WSGG method originally proposed by Song and Viskanta [4]. They recognized the difficulty which arises in WSGG modelling of gas radiation in non-uniform media. They noted that spectral integration of monochromatic RTE over the gray gas wavenumber intervals yields appearance of numerous so-called Leibnitz terms. These terms arise in the integration because the boundaries of the gray gas wavenumber intervals vary with spatial location in non-uniform media if the supplemental cross-sections defining the gray gas wavenumber intervals are fixed [2–6]. Simple neglect of these additional terms can yield significant error in the prediction of radiative transfer. It was proposed by Song and Viskanta that the gray gas spectral intervals be fixed to avoid the appearance of the Leibnitz terms by varying the supplemental absorption cross-sections in such a way that the spectral intervals obtained at some chosen reference thermodynamic state are maintained the same at all local states in the medium. However, the approach was not implemented in the original paper, nor in the following applications of the WSGG method [7]. Despite this issue, the WSGG method continues to be attractive to researchers, and further improvement of the method has been developed recently [8–12]. The different versions of the WSGG method are distinguished mainly by the difference in mathematical representations of the gray gas absorption coefficients and the corresponding weights. For example, recent work has proposed the determination of coefficients in the

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Nomenclature

a	gray gas weights
C	gas absorption cross-section [m^2/mol]
E	radiation emissive power [W/m^2]
F	ALBDF
I	radiation intensity [W/m^2/sr]
L	gas layer thickness [m]
ℓ	ℓ -distribution
N	gas molar density [mol/m^3]
p	gas total pressure [atm]
s	path variable [m]
T	gas temperature [K]
u	scaling coefficient
Y	gas specie mole fraction
Greek symbols	
β	ℓ -distribution overlapping parameter
ϕ	symbolic notation for given gas thermodynamic state, $\phi = \{T, Y, p\}$
ξ	continuous variable as a limit of a discrete absorption cross-section
κ	gas absorption coefficient [m^{-1}]
μ	moment
η	wavenumber [cm^{-1}]
σ	Stefan-Boltzmann constant, 5.67×10^{-8} W/m^2/K^4
τ	transmissivity
Subscripts	
b	blackbody
j	gray gas number
n	number of gray gases in the model
P	Planck mean
R	Rosseland mean
s	order of the moment
loc	local
ref	reference

WSGG model through the (*i*) superposition of correlations for a mixture of gas species and soot, and (*ii*) superposition of the separate coefficients for each specie (and soot) [9]. These correlations are obtained from the high resolution spectral database HITEMP-2010 [13] by fitting emittance data for a range of thermodynamic conditions. The examples considered in [11,12] demonstrate satisfactory accuracy for non-isothermal, non-homogeneous cases.

The classical WSGG method does not possess the flexibility of the SLW method which, unlike the WSGG method, can be used with an arbitrary number of gray gases and any gas compositions. However, although the WSGG method may not be sufficiently accurate to provide benchmark results, its robustness, consistency and computational efficiency make it a viable alternative to more sophisticated models, especially in applications in which thermal radiation is only one part of several other complex, coupled modeled phenomena.

Approaches in the SLW methods are based on assumptions of correlated/scaled absorption spectra which can be reduced to the assumption on which separation of dependence of gas absorption coefficient on space variable (and therefore, on the local gas thermodynamic state) and on wavenumber. The usual approaches are based on specification of a reference thermodynamic state and construction of the associated SLW reference histogram absorption spectrum at this state. The local histogram spectrum at other spatial locations is then scaled to the reference spectrum by a scaling coefficient which can be different for different gray gases (correlated models), or by a single coefficient (scaled models). Because this assumption, in general, is not valid for real spectra, accuracy of the predictions depends critically on the choice of reference state. The reference gas temperature T_{ref} has been chosen from some physical consideration which has included the maximum temperature in the system, the minimum temperature in the system, the volume-averaged temperature, the Planck mean temperature, and the emission-weighted temperature [6,14].

Application of the correlated spectrum assumption does not necessarily provide optimal accuracy of prediction of radiative quantities. For systems with large temperature and/or concentration gradients, the dependence of the spectral properties on these properties can produce significant errors. One way to solve this temperature dependence problem is to model the absorption coefficient's dependence on the temperature as was done in the Multi-Group Full Spectrum *k*-distribution Method (MGFSK) [15]. The MGFSK Method provides good results for inhomogeneous media, and the accuracy may be increased by increasing the number of groups. However, the method remains approximate. A more accurate but more computationally expensive alternative is presented by Tencer and Howell [14]. Their multi-source full spectrum *k*-distribution method is capable of providing exact results for one-dimensional geometries with piecewise constant temperature and absorption coefficient in the same way that the full spectrum *k*-distribution method is able to provide exact results for homogeneous media. The approach as outlined in [14] provides a significant increase in accuracy when compared to other *k*-distribution based approaches while remaining significantly less computationally expensive than a standard line-by-line solution. The errors introduced by this method are entirely due to the multilayer approximation. As a result, the method converges linearly as the number of layers is increased. This allows the error relative to the line-by-line solution to be estimated.

The need for simple, accurate engineering approaches for the prediction of radiation transfer in non-uniform high temperature gas applications remains, particularly in comprehensive reacting flow modeling applications. Recent progress in SLW modelling, including the development of the Generalized SLW model [2] and the Rank Correlated SLW model [3,16], opens new opportunities for a model developed independently on the assumption of a scaled spectrum. The scaled spectrum assumption has never been explored in the framework of the SLW model, but it has potential to improve performance of the SLW method when correlated models fail. The scaled SLW models which are outlined here are not intended to replace the correlated SLW models developed earlier, but rather to complement the correlated SLW model.

In the scaled spectral model it is assumed that the absorption cross-sections of a particular gas or gas mixture at two different local thermodynamic states denoted symbolically as $\phi_1 = (T_1, Y_1, p)$ and $\phi_2 = (T_2, Y_2, p)$ are related by a simple scaling for the entire spectrum (for all wavenumbers $\eta > 0$) as

$$C_\eta(\phi_2) = u(\phi_1, \phi_2) C_\eta(\phi_1)$$

The scaled spectrum assumption is a particular and more restrictive case of the correlated spectrum assumption. However, application of the scaled model may have some advantages. Correlated spectrum models rely on how well real spectra are correlated but they cannot overcome the fact that real spectra are never completely correlated. Consequently, in the case of highly non-uniform media the prediction of radiative transfer can be inaccurate. Scaled spectral models may improve performance with the help of a more informed choice of scaling coefficient $u(\phi_1, \phi_2)$ for better representation of gas absorption spectra at the local thermodynamic state in the prediction of radiative transfer in non-uniform media. Further, the scaled model may have the advantage of being simpler in its construction and implementation.

Once the scaling coefficient is determined, the scaled SLW model can be readily constructed. Therefore, the critical element