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Multiple scattering contribution to the diffuse light of a night sky: A model which embraces all orders of scattering

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ABSTRACT

The mechanism in which multiple scattering influences the radiance of a night sky has been poorly quantified until recently, or even completely unknown from the theoretical point of view. In this paper, the relative contribution of higher-scattering radiances to the total sky radiance is treated analytically for all orders of scattering, showing that a fast and accurate numerical solution to the problem exists. Unlike a class of ray tracing codes in which CPU requirements increase tremendously with each new scattering mode, the solution developed here requires the same processor time for each scattering mode. This allows for rapid estimation of higher-scattering radiances and residual error that is otherwise unknown if these radiances remain undetermined. Such convergence testing is necessary to guarantee accuracy and the stability of the numerical predictions.

The performance of the method developed here is demonstrated in a set of numerical experiments aiming to uncover the relative importance of higher-scattering radiances at different distances from a light source. We have shown, that multiple scattering effects are generally low if distance to the light source is below 30 km. At large distances the multiple scattering can become important at the dark sky elements situated opposite to the light source. However, the brightness at this part of sky is several orders of magnitude smaller than that of a glowing dome of light over a city, so we do not expect that a partial increase or even doubling the radiance of otherwise dark sky elements can noticeably affect astronomical observations or living organisms (including humans). Single scattering is an appropriate approximation to the sky radiance of a night sky in the vast majority of cases.

1. Introduction

Artificial light at night is known to spread over a considerable distance [7,12] thus having a serious impact on nature and human lives (see e.g. [27]). It is well recognized that the number of photons reaching a distant place is generally low, but the rate at which the ground illuminance decreases with increasing distance strongly depends on many factors including atmospheric optical properties [20,32], spectra and/or angular emission patterns of artificial lights [2,29], blocking obstacles [14,28] and terrain [6,11], ground albedo [25], etc. Some of the above parameters and functions can be determined experimentally at the site, while others are difficult to obtain without applying complex analysis, theories or intensive computations. For instance, the cumulative upward light emissions are normally controlled by the physical characteristics of dominant city lights that are usually distributed non-uniformly within an urban area [22].

However, a few effects still remain unsatisfactorily treated. Among the wide range of factors that are associated with skyglow, the multiple light scattering currently has an unknown influence on forming the diffuse light of a night sky. The contribution of higher scattering orders to the light field is poorly quantified and has emerged as a substantial source of uncertainty in numerical modeling and night light characterization. The role of multiple-scattering in skyglow modeling is sometimes overestimated or underestimated, thus potentially resulting in a number of misinterpretations or unnecessarily complex measures in solving some particular problems (e.g. in assessing the ultimate levels of sky glow in different locations).

The early skyglow models [9,15,17] proved to be very useful in rapid estimation of clear-sky radiance or luminance distributions. A new generation of skyglow model that benefits from a wide range of possible applications and easy use under various atmospheric conditions has been developed only recently (see [21]). The model is scalable and designed for realistically shaped emitters (e.g., cities or defined land surface areas) and incorporates 3D-effects in a consistent manner [19].

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The importance of simple, fast, and accurate algorithms amplified immediately after a need for optimum design of public lighting systems escalated worldwide during the past decade. One of the key goals is to perform time-efficient evaluation of the optical effects related to different lamp spectra or different lamp distributions. An estimation of the typical changes to sky glow occurring as a result of street lighting conversions would not have been possible without the efficient algorithms that allow for mass computations or other real-time data analysis [18].

Introducing higher scattering orders into computational models could make the numerical predictions inherently more accurate, but such numerical modeling usually needs long computing time, especially if Monte Carlo [24] or associated methods [3] are in use. No doubt the best strategy is to improve accuracy for a reasonable computing time, i.e. to find a tradeoff between accuracy achievements and CPU requirements. For instance, Bará et al. [4,5] have reduced the computational time by using an alternative modal approach expanding the night-sky radiance in terms of 2D orthogonal functions.

However, due to its simple concept and easy programming, the method of successive orders of scattering (SOS) still remains the most popular tool to solve the radiative transfer equation. The convergence of SOS is generally fast in a thin atmosphere (or at short optical paths), but it is worsened as the eigenvalues increases [16]. Basically, the *n*th scattering order radiance is a triple integral of the (*n*-1)th radiance field: the theoretical model includes the integration over all azimuth and zenith directions as well as over the whole atmospheric volume. Nevertheless, obtaining the radiance field in a nocturnal environment is a 4D problem because the night-sky radiance also depends on the horizontal distance to a light source, even if the model of a horizontally-homogeneous atmosphere is taken into consideration. If inadequately designed the computational model would not converge within a given numerical accuracy and together with large demands on CPU time would represents a major weakness that could make ray-tracing and related methods inefficient. Specifically, the cumulative effect of many finite errors introduced at each single computation may become large if e.g. an increment is chosen improperly for a specific integration variable. Starting with the second scattering order the cumulative error can significantly affect the convergence of the numerical schema, because, unlike the single-integral form for the first-order scattering, the double (and any higher) scattering radiances are quadruple integrals. That is the reason an otherwise small imperfection in single-scattering radiance computation could grow considerably with each higher scattering order. Exceeding an acceptable error tolerance may cause an uncontrolled growth of radiance computed for some *i*th and subsequent scattering orders, implying that the total radiance (= the sum of all scattering orders) would grow above all limits. This is a weak point of insufficiently inspected computations in which the series are truncated at e.g. second scattering order without validating the global convergence of the higher scattering orders. A typical consequence is that the radiance data computed in this way are partly or even highly overestimated depending on threshold error value and convergence criteria for a given computational model. A major shortcoming of a truncated series is that each higher-scattering radiance is a finite value, so it is difficult or rather impossible to uncover any potential convergence problems when limiting the computation to a few (e.g. first two) scattering orders.

We need an analytical model that is always equally accurate and provides a facility for computing all radiance components. An analytical model can predict radiance even under extremely high or low turbidity conditions and can be used to construct theoretically well-founded approximations that are otherwise impossible to obtain by applying e.g. ray-tracing methods. To-date no analytical solution which embraces all orders of scattering exists and the contributions of high-scattering components to the night-sky radiance is completely unknown. No quantification of the phenomena exists, no numerical experiments have been performed, and the theoretical development is in its infancy. This is a wholly new field.

It is a goal of this paper to derive and validate a comprehensive night-sky radiance model for all orders of scattering. Such a model includes the Rayleigh theory and aerosol optics in a stratified atmosphere in which both scattering and true absorption are spectrally-dependent. The radiance field at arbitrary altitude or horizontal distance from a light-emitting source can be calculated for a non-uniform surface albedo ranging (theoretically) from 0 to 1.

This is the first time the relative contributions of higher-scattering radiance to the total sky radiance are quantified and applied to different situations. It is our intention to uncover the relative importance of higher-scattering radiance components at different distances from a light source, under different turbidity conditions, and in distinct spectral bands. Our aim is to estimate the magnitude in which each factor influences the radiance in different parts of sky.

The new model that fully accounts for all orders of scattering can be used to draw firm conclusions about the range of applicability of single-scattering approximation in skyglow modeling, and to identify situations in which multiple scattering effects cannot be ignored.

2. Analytical solution to the all orders of scattering

Solving the radiative transfer in the night atmosphere illuminated from below is a nontrivial problem, because the short distance to the light source makes the far-field approximations largely inapplicable. The geometry of light scattering in the atmospheric environment is shown in Fig. 1, where a light-emitting source in point *S* is situated at the ground. Radiance computations are made for the measuring point (*O*), with the horizontal distance to the light source is *R* and the altitude above the ground level is *h*.

In the chosen coordinate system both the atmospheric optical thickness (τ) and the altitude (h) are measured along z-axis, where τ decreases as h increases. The optical thickness reaches its maximum value (τ_0) at the ground (i.e. at $h = 0$), while τ is zero at the top of the atmosphere. The theoretical radiance of a clear sky is a superposition of all elementary optical signals along the direction of sight $\mathbf{\Omega} = (\mu, A)$. Here $\mu = \cos z$; z and A are zenith and azimuth angles, respectively. A small volume element at the altitude $h'(\tau')$ is illuminated from all directions $\mathbf{\Omega}' = (\mu', A')$ and scatters the light towards $\mathbf{\Omega}$. The total radiance field involving scattering from all atmospheric layers can be obtained by solving the complex radiative transfer equation, the integro-differential equation (see e.g. [10]). However, the basic theories suggest that the multiple scattering component drops as quickly as the optical path length decreases, so the contribution from light experiencing many scattering events to the total radiance is expected to be low if *R* is short and observations are made in a cloud-free atmosphere. In such a case the SOS is a preferred approach to modeling sky radiance because of the simple concept and inexpensive computation, especially if confined to just one scattering order. SOS is to decompose the radiance into a sum of all scattering orders, while obtaining rapidly convergent series. The principal question is how many scattering orders (modes) we really need to simulate night-sky radiance within a specified error tolerance.

SOS only requires the single scattering radiance to be determined as a generating field and all subsequent radiance-components are computed recursively. On one hand, the amount of electromagnetic radiation entering the atmosphere at its top is negligibly small (nearly zero). On the other hand, the light escaping the surface strongly depends on downward radiative flux and