Journal of Quantitative Spectroscopy & Radiative Transfer 206 (2018) 233–241

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

On the regularization for nonlinear tomographic absorption spectroscopy

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ARTICLE INFO

Article history:
Received 27 July 2017
Revised 15 November 2017
Accepted 17 November 2017
Available online 21 November 2017

Keywords:
Absorption spectroscopy
Nonlinear tomography
Combustion diagnostics

ABSTRACT

Tomographic absorption spectroscopy (TAS) has attracted increased research efforts recently due to the development in both hardware and new imaging concepts such as nonlinear tomography and compressed sensing. Nonlinear TAS is one of the emerging modality that bases on the concept of nonlinear tomography and has been successfully demonstrated both numerically and experimentally. However, all the previous demonstrations were realized using only two orthogonal projections simply for ease of implementation. In this work, we examine the performance of nonlinear TAS using other beam arrangements and test the effectiveness of the beam optimization technique that has been developed for linear TAS. In addition, so far only smoothness prior has been adopted and applied in nonlinear TAS. Nevertheless, there are also other useful priors such as sparseness and model-based prior which have not been investigated yet. This work aims to show how these priors can be implemented and included in the reconstruction process. Regularization through Bayesian formulation will be introduced specifically for this purpose, and a method for the determination of a proper regularization factor will be proposed. The comparative studies performed with different beam arrangements and regularization schemes on a few representative phantoms suggest that the beam optimization method developed for linear TAS also works for the nonlinear counterpart and the regularization scheme should be selected properly according to the available *a priori* information under specific application scenarios so as to achieve the best reconstruction fidelity. Though this work is conducted under the context of nonlinear TAS, it can also provide useful insights for other tomographic modalities. © 2017 Elsevier Ltd.

1. Introduction

Absorption spectroscopy has found extensive applications for gas sensing due to its advantages such as species-specificity, ease of implementation, and high sensitivity [1–5]. For example, it has been used for flame measurements in which temperature and concentration of the absorbing species can be simultaneously obtained [6]; for environmental monitoring of green-house gas emission such as CH_4 and CO [7]; and for health diagnostics by analyzing the breathing air [8], to name a few. A few excellent reviews are available on absorption spectroscopy for gas sensing [1,9]. Despite these successful examples, absorption spectroscopy suffers from a critical drawback which prevents its further applications. This drawback originated in its line-of-sight (LOS) nature which limits its applicability to scenarios where the thermodynamic property of the absorbing gas is uniform along the LOS. Nevertheless, such condition is not necessarily satisfied in some scenarios *e.g.* in turbulent flames both the chemical compositions and temperature vary from place to place [10,11].

To overcome this limitation *i.e.* to enable the spatial resolution of the LOS absorption spectroscopy, enormous research efforts have been invested during the past decades [12]. It has been reported by independent groups that 1D spatial resolution can be enabled when multispectral absorption information was utilized [13,14]. Further, by combining tomography with absorption spectroscopy, 2D/3D spatial resolution can be enabled leading to an imaging technique called tomography absorption spectroscopy (TAS) [5,12,15–20]. Depending on the mathematical principle it relies on, TAS can be further divided into linear and nonlinear modalities respectively. The linear TAS is based on the principle of classical tomography which originated from medical imaging [21], and the measured projections are the LOS integrals of the target field *e.g.* the field of absorption coefficient for a specific absorption transition [22,23]. On the other hand, the nonlinear TAS is based on the recently proposed concept of nonlinear tomography [17], and was numerically proven to be capable of simultaneously

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https://doi.org/10.1016/j.jqsrt.2017.11.016
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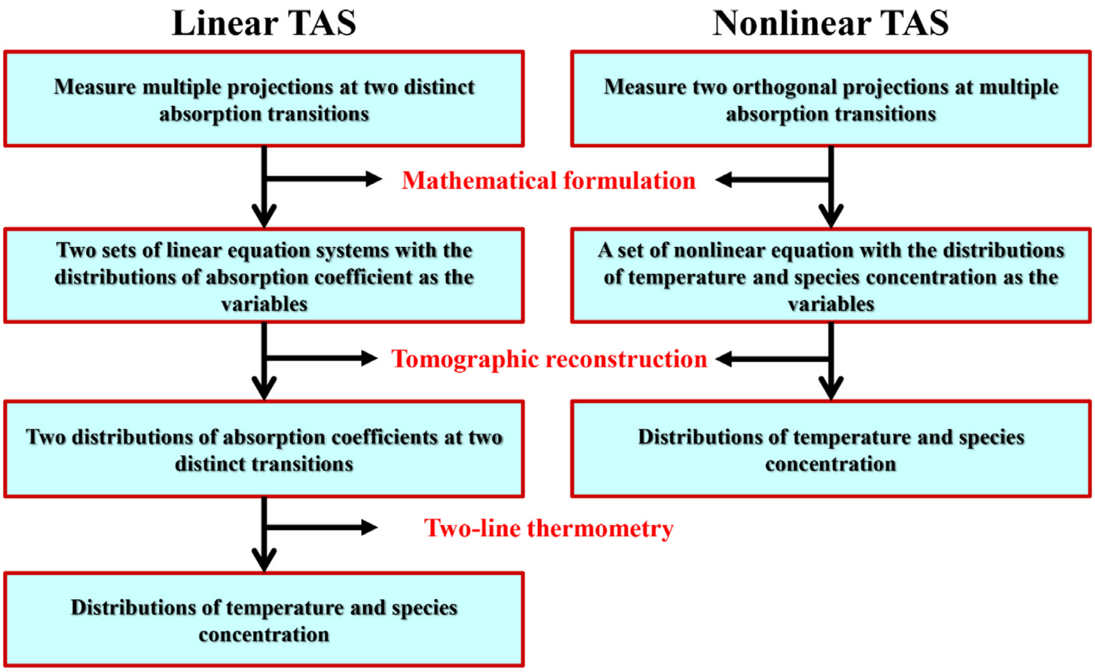

Fig. 1. The flow charts that compare the tomographic procedures of both linear and nonlinear TAS.

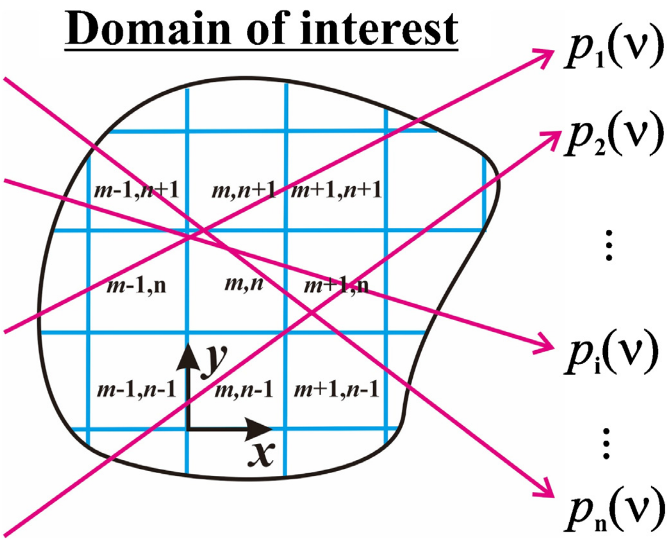

Fig. 2. The schematic for illustration of the mathematical formulation of nonlinear TAS. The domain of interest is discretized into square pixels and the probing beams are plotted as magenta lines.

recovering multiple fields of physical quantities such as temperature, species concentration, and pressure [16]. In nonlinear TAS, the measured projections are nonlinearly dependent on at least one of the target fields. For example, the LOS-integrated absorbance has strong and weak nonlinear dependencies on temperature and pressure profiles respectively.

Fig. 1 compares the implementation procedures between the linear and nonlinear TAS in the previous demonstrations [20,24,25]. In linear TAS, typically multiple projections at two absorption transitions are measured; through mathematical formulation, two sets of independent linear equation systems can be obtained with the distributions of absorption coefficients at each of the transitions as the variables; the tomographic reconstruction for each transition will lead to two maps of absorption coefficients; and finally by performing the so-called two-line thermometry [12] for each grid point, the distributions of both temperature and species concentration can be determined. On the contrary, nonlinear TAS typically measures only two orthogonal projections but at multiple transitions [26]; through the practice of nonlinear tomography, a set of nonlinear equations can be obtained with the distributions of both temperature and species concentration as the variables; and the solution of the nonlinear equation system directly leads to the distributions of temperature and species concentration [27,28].

As can be seen from this comparison that there is a major advantage of nonlinear TAS over the linear version: when the same number of projections are used in the experiment, much richer spectral information can be incorporated into the tomographic reconstruction since much more equations are available for nonlinear TAS. The number of nonlinear equations scales linearly with the number of absorption transitions used. This feature makes nonlinear TAS more favorable than the linear modality as the former one can achieve similar reconstruction quality but with greatly reduced number of projections than the latter one [15].

So far, all the previous numerical/experimental demonstrations of nonlinear TAS have been demonstrated with only two orthogonal projections [12]. However, it has to be pointed out that nonlinear TAS is not limited to such beam arrangement. It can be implemented with arbitrary beam configuration that has been adopted for the linear modality. Recently, Yu *et al* successfully proposed and validated a beam optimization method for linear TAS [19]. Nevertheless, this method has not been tested yet under the context of nonlinear TAS. Thus, one purpose of this work is to examine the performance of nonlinear TAS using other beam arrangements and test the effectiveness of the beam optimization technique that has been developed for linear TAS.

In contrast to linear TAS in which a span of regularization techniques are available, so far, only the smoothness condition of the target fields was used as a prior for regularization [29]. However, there are also other priors such as sparseness and model-based priors that are commonly available but have not been explored to aid the inversion of a nonlinear TAS problem. Thus, the other goal of this work is to systematically examine a few representative reg-