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Inverse kinematics of five-axis machines near singular configurations

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Abstract

In five-axis milling, singular configurations of the machine axes may cause tool path errors or collisions between the tool and parts of the milling machine. This paper presents an algorithm for calculating the inverse kinematics of five-axis machines close to singular configurations. The algorithm modifies the exact inverse kinematics in order to give robustness to singularities at the expense of a small tool orientation deviation. The kinematics of a five-axis machine with non-orthogonal rotary axes is analyzed. The forward kinematics is developed, and a closed form solution of the inverse kinematics is presented. The kinematics and the singularity algorithm are implemented in a postprocessor, and machining tests are conducted to verify the algorithms.

Keywords: Postprocessor; Inverse kinematics; Five-axis milling; Singularity

1. Introduction

The kinematics of five-axis machines has captured the attention of many researchers for several years. While postprocessing cutter location (CL) data for a conventional three-axis machine is a geometrically simple task, post-processing for a five-axis machine is more complex due to the rotary axes. The problems are related to solutions for forward and inverse kinematics, the linearization of the tool path, and the positioning of the machine axes near a kinematic singularity. Today, postprocessing is usually performed off-line as a part of the NC program preparation. In the future we will probably see a stronger tendency towards real-time postprocessing on the machine control units [1], which increases the demand for fast and robust calculation of the inverse kinematics.

Takeuchi and Watanabe [2] presented principles for linearization and feed rate control. They also discussed control of the spindle speed, based on varying tool/workpiece contact point. Forward and inverse kinematics for different kinds of five-axis machines have been presented and discussed in many papers, for example, by Lee and She [3]. Kinematic chain design and the usability of different kinds of five-axis machines are discussed by Bohez [4] and Tutunea-Fatan and Feng [5].

A common method for avoiding problems near the singular configuration of five-axis machines is to retract the tool along the tool axis [6]. This method will cause interrupts in the tool path and is therefore undesirable in simultaneous five-axis machining. The problem of singular configuration is discussed by Affouard et al. [1] who developed a method for avoiding the machine singularity through a tool path planning algorithm in the CAD/CAM system. Another method for reducing the machining error near singularities, by optimizing the sequence of machine-axis rotations, has been proposed by Munlin et al. [7].

Most of the literature to be found on inverse kinematics for five-axis machines focuses on machines with orthogonal rotary axes. This paper presents the forward kinematics and closed form solution for the inverse kinematics for a five-axis machine with non-orthogonal rotary axes, e.g. the Deckel Maho DMU 50 eVolution. Linearization of the tool path is discussed, and a new algorithm for positioning the machine axes close to the singular configuration is presented.

2. Forward and inverse kinematics

Inverse kinematics is used to determine the set of axis variables (e.g. X, Y, Z, B, and C) that will produce the

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desired CL (x, y, z, i, j, and k) given in the CL data file. Postprocessing primarily utilizes the inverse kinematics. The forward kinematics is used to calculate the CL from the machine axis variables. This is used in postprocessors for checking tool path accuracy in the linearization algorithm.

2.1. The DMU 50 eVolution

A photo of the rotary table of the DMU 50 eVolution five-axis milling machine is shown in Fig. 1, and a sketch of the side view of the machine is presented in Fig. 2. A special feature of the machine is that the rotary axes (B axis and C axis) are non-orthogonal. The angle between the two axes is 45°. The center axes of the two rotary axes cross in a point at a vertical distance d from the fixed coordinate frame $x_0y_0z_0$. The working range of the B axis is $[0°, 180°]$, and the working range of the C axis is $(-\infty, \infty)$. In the NC file the C-axis position is given with the range $(-180°, 180°]$. Using the B and C axes, every tool orientation perpendicular to the surface of the half of a sphere can be achieved.

2.2. Forward kinematics

The following coordinate frames are defined to derive the kinematic equations for the machine:

- $x_0y_0z_0$: The base coordinate frame, located in the center of the table surface when $B = C = 0°$.
- $x_1y_1z_1$: A translation of $x_0y_0z_0$ at a distance d along z_0.
- $x_2y_2z_2$: A rotation of $x_1y_1z_1$ at an angle $+45°$ around x_1. Frame $x_0y_0z_0$, frame $x_1y_1z_1$ and frame $x_2y_2z_2$ are fixed; they do not move with the machine axes.

Fig. 1. Non-orthogonal rotary axes in a milling machine.

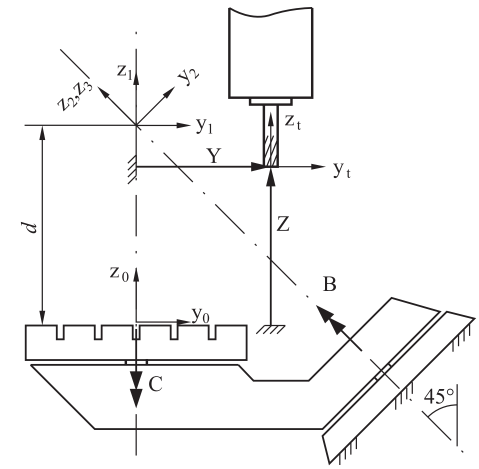

Fig. 2. Side view sketch of a five-axis machine with non-orthogonal rotary axes. Shown with the B axis in $B = 0°$ position.

- $x_3y_3z_3$: A rotation of $x_2y_2z_2$ at an angle B around z_2.
- $x_4y_4z_4$: A rotation of $x_3y_3z_3$ at an angle $-45°$ around x_3.
- $x_5y_5z_5$: A translation of $x_4y_4z_4$ at a distance $-d$ along z_4. The frame $x_5y_5z_5$ is always located at the center of the machine table, also after the B axis has been rotated.
- $x_wy_wz_w$: The workpiece coordinate frame. Obtained by rotating $x_5y_5z_5$ at an angle $-C$ around z_5.
- $x_ty_tz_t$: A frame fixed to the milling tool with the origin at the tool tip.

A homogenous matrix that transforms the coordinates of a point from frame $x_ny_nz_n$ to frame $x_my_mz_m$ is denoted by T_m^n. By using the convenient shorthand notation $\sin\phi = s_\phi$ and $\cos\phi = c_\phi$ the transformation matrices for the frames defined above are written as follows:

$$T_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (1)$$

$$T_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{45°} & -s_{45°} & 0 \\ 0 & s_{45°} & c_{45°} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (2)$$

$$T_2^3 = \begin{bmatrix} c_B & -s_B & 0 & 0 \\ s_B & c_B & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (3)$$

$$T_3^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{45°} & s_{45°} & 0 \\ 0 & -s_{45°} & c_{45°} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (4)$$