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A physically based gradient plasticity theory

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Abstract

The intent of this work is to derive a physically motivated mathematical form for the gradient plasticity that can be used to interpret the size effects observed experimentally. The step of translating from the dislocation-based mechanics to a continuum formulation is explored. This paper addresses a possible, yet simple, link between the Taylor's model of dislocation hardening and the strain gradient plasticity. Evolution equations for the densities of statistically stored dislocations and geometrically necessary dislocations are used to establish this linkage. The dislocation processes of generation, motion, immobilization, recovery, and annihilation are considered in which the geometric obstacles contribute to the storage of statistical dislocations. As a result, a physically sound relation for the material length scale parameter is obtained as a function of the course of plastic deformation, grain size, and a set of macroscopic and microscopic physical parameters. Comparisons are made of this theory with experiments on micro-torsion, micro-bending, and micro-indentation size effects.

Keywords: Gradient plasticity; Material length scale; SSDs; GNDs; Size effects; Micro-bending; Micro-torsion; Micro-indentation

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1. Introduction

The problem in developing a macroscopic model embedded with a micromechanical-based theory of inelasticity which could be used as an engineering theory for both the analysis and in computer-aided design of materials is a topical and still unsolved material science problem. Attempts to construct such a theory are faced with the difficulties in describing the microscopic structure of materials in terms of macroscopic mechanics. On the other hand, at the present time, it is still not possible to perform quantum and atomistic simulations on realistic time scale and structures. When load is applied, the inelastic deformation that occurs in most cases is not homogeneous, but reveals fluctuations on various length scales. This heterogeneity plays a key role in determining the macroscopic properties of materials. A physically based theory that bridges the gap between the conventional continuum theories and the micromechanical theories should be developed as a remedy for this situation.

Material length scales (i.e., the dependence of mechanical response on the structure size) are of great importance to many engineering applications. Moreover, the emerging areas of micro and nanotechnologies exhibit important strength differences that result from continuous modification of the material microstructural characteristics with changing size, whereby the smaller is the size the stronger is the response. There are many experimental observations which indicate that, under certain specific conditions, the specimen size may significantly affect deformation and failure of the engineering materials and a length scale is required for their interpretation. Experimental work on particle-reinforced composites has revealed that a substantial increase in the macroscopic flow stress can be achieved by decreasing the particle size while keeping the volume fraction constant (Lloyd, 1994; Rhee et al., 1994; Zhu and Zbib, 1995; Nan and Clarke, 1996; Kiser et al., 1996). A similar strengthening effect associated with decreasing the diameter of thin wires in micro-torsion test has been reported by Fleck et al. (1994) and with decreasing the thickness of thin beams in micro-bending test has been reported by Stolken and Evans (1998), Shrotriya et al. (2003), and Haque and Saif (2003). Moreover, micro- and nano-indentation tests have shown that the material hardness increases with decreasing indentation size (e.g., Stelmashenko et al., 1993; DeGuzman et al., 1993; Ma and Clarke, 1995; Poole et al., 1996; McElhaney et al., 1998; Lim and Chaudhri, 1999; Elmustafa and Stone, 2002; Swadener et al., 2002). Indentation of thin films shows an increase in the yield stress with decreasing the film thickness (Huber et al., 2002). An experimental work by Taylor et al. (2002) shows an increase in the flow stress with decreasing hole size for geometrically similar perforated plates under tension, i.e., plates with a hole or several holes. Furthermore, there are many other well-known problems that show strong size effects. One example is the testing of polycrystalline materials which shows an increase in both yield and flow stresses, or equivalently the hardness, with decreasing the grain diameter; the so-called Hall-Petch behavior. These experiments have, thus, shown increasing in strength with decreasing size at the micron and submicron scales where the representative length scale ℓ of the deformation field sets the qualitative and quantitative behavior of the size effect.