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Random uncertainty propagation in estimates of sphere parameters from coordinate measurements

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Abstract

Measurements of reference spheres are widely used in verification and calibration of coordinate measuring systems. In this paper, a model for the propagation of random errors in the estimation of sphere parameters is outlined. The dependence of the error propagation on sampling area and sample density is demonstrated using both simulated and measured data. The model can be used to characterise the random error of a coordinate measurement system. It can also be used to determine the error limits on a single sphere parameter estimate or to determine the required sampling strategy to ensure that the confidence limits on an estimate remain within predetermined bounds.

Keywords: Error propagation; Coordinate metrology; Measurement uncertainty; Calibration

1. Introduction

Coordinate metrology has become a key component of industrial measurement and is routinely used in process monitoring and quality inspection [1]. With the increasing requirement for the measurement of the form and texture of surfaces with micro- and nano-scale accuracy, high-precision surface profilers have been developed using the coordinate measurement principles [2], such as the Micro-Measure Plus series [3], the Form Talysurf series [4], and TaiCann XYRIS series [5], etc.

Measurements of reference spheres are widely used in verification and calibration of coordinate measuring machines and their probing systems [6–8]. They are also used for calibration of the surface profilers. Hill et al. [9] have developed a technique for calibration of the orthogonal errors between the axes of a non-contact high-precision surface profiler by analysing the sphericity and the deviation of the estimated radius of a measured reference sphere. Mills [10] has patented a technique for calibrating and compensating the disorientation of the axes of the Form Talysurf surface profiler by analysing the residuals and the central coordinates of the intersectional circles from the measurement of a reference sphere.

All measurement systems not only suffer from the systematic errors, such as the axis alignment and positioning errors, but also some degree of the inherent random error caused by non-constant sources of error either in time or in space [2]. Under certain circumstances, relatively small random errors can have a significant effect on the estimation of geometric parameters based upon those measurements. This paper examines the relationship between sampling strategy and random uncertainty in sphere radius estimation for the three-dimensional measurement systems, allowing appropriate choices to calibrate the point-to-point measurement error of the measurement technique employed.

2. Background

2.1. Propagation of errors

From the Guide to the Expression of Uncertainty in Measurement (GUM) [11], the measurand Y is dependent

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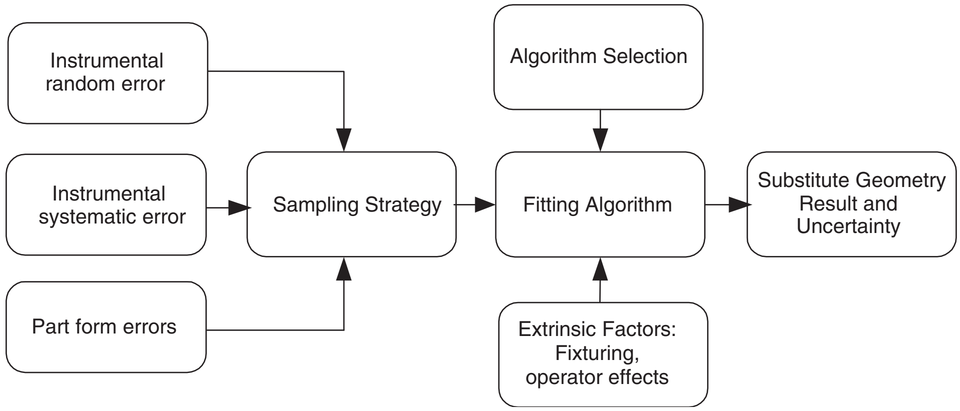

Fig. 1. Various factors affecting the measurement uncertainty.

upon a number of different sources of uncertainty X_i ($i = 1$–N).

$$Y = f(X_1, X_2, \ldots, X_N), \tag{1}$$

$$u_c^2 = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j). \tag{2}$$

In Eq. (2), the measurement uncertainty is the summation of each individual uncertainty source. In determination of measurement uncertainty, the difficulty is in quantifying the sources of uncertainty and the sensitivity coefficients in Eq. (2). One general paradigm for separating uncertainty sources of a coordinate measurement system, as depicted by Phillips [7], is shown in Fig. 1.

For the research outlined in this paper, which aims to investigate the random uncertainty on the estimated radius of a sphere based upon 3D coordinate measurements, many factors shown in Fig. 1 can be neglected and the relationship between the factors is clarified in the following discussion.

Various algorithms for spherical form evaluation have been proposed and used, including the linear least-squares (LLS) method, the non-linear least-squares (NLS) method, the minimum zone (MZ) method, the minimum average deviation (MAD) method, and more recently the error curve analysis method [12]. It has been noted [13] that the least-squares method is appropriate where random measurement errors predominate because it produces the smallest combined uncertainty in the results, and currently, is used almost universally.

Zhang [14] tested four popularly used spherical form evaluation algorithms: the LLS method, the NLS method, the MAD method and the MZ method. The tests showed that the NLS method produces the lowest measurement uncertainty of the estimated radius wherever random errors or form errors are predominate. Most importantly, the form errors of the measured spherical surfaces do not change the random measurement uncertainty of the estimated radius derived by the NLS method. However, if the other three fitting methods are applied, the random measurement uncertainty varies when the measured sphere have difference form errors.

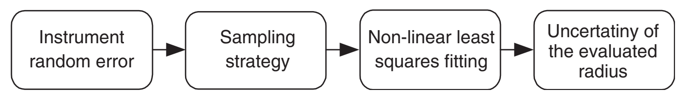

Fig. 2. Factors affecting the measurement results in case under consideration sphere.

Systemic errors of the CMSs produce a deformation on the measured data, which is regarded as similar to the effects of the form errors [9]. Therefore, the form errors of the measured sphere and the systematic errors of the measuring instruments will not be considered as factors that change the random uncertainty of the estimated radius when the NLS method is used.

Extrinsic factors can be carefully controlled to minimise their effect on the measurement results [15]. With all these factors defined, the flow chart in Fig. 1 can be simplified to that shown in Fig. 2.

Fig. 2 demonstrates that the measurement random uncertainty of the estimated radius of a measured sphere is reduced to a dependence on the instrumentation random error and the sampling strategy using the NLS sphere fitting method. That is, the random measurement uncertainty of the estimated radius of a measured sphere derived by the NLS method is only a function of the sampling size, sampling density and the random measurement error. Thus, examining and revealing the function between the uncertainty and the sampling strategy will allow a reliable prediction of the random error of a coordinate measuring system.

2.2. Non-linear least squares sphere fitting

NLS sphere fitting [16,17] derives the parameters using a strict least squares definition, minimising the error function shown in Eq. (3). Iteration is used and the algorithm is terminated once the required accuracy for the