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Dynamics of the rolling and sliding of an elongated cylinder

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ABSTRACT

A snap of a finger on an elongated cylinder produces on it a surprising fast spinning motion during which its mass center rises with very large oscillations. After that, the mass center goes to a long quasi-stationary oscillations state and eventually goes down very slowly. To explain this behavior we present a theoretical and numerical analysis of the dynamics of a spinning elongated cylinder moving with a single point of contact on a horizontal plane under the action of gravity. The study has been made taking into account the rolling and sliding dissipation as well as the Kutta–Joukowski airflow effect. The results of the simulations are in agreement qualitatively with the observed real motion.

1. Introduction

The origin of this paper was the observation of the surprising fast spin motion that acquires an AA battery after a snap of a finger (click on the figure below to see a slow motion video of the initial instants of the motion, or see the video on youtube: www.youtube.com/watch?v=4LYv4NsiliQ). Video S1 As in the amazing tippe top or in the spinning egg (see [1–4]), we observe that the mass center of the battery rises with a few very large oscillations, after what the battery goes to a quasi-stationary state while the mass center goes down slowly. To explain this behavior, we have studied the motion of an elongated cylinder with a single point of contact on a horizontal surface under the action of gravity.

This problem is a special case of the motion of a rigid body of revolution over a plane, which is a classical mechanics problem that was studied in the past by Routh [5] and Appell [6]. In particular, the motion of a disk has been widely studied (see for instance [7–16]). In the non-dissipative case the non-holonomic constrained motions of both disk and cylinder are integrable and, using this fact, several authors have studied their stability as well as the associated bifurcations [7–9]. The dissipative case for a disk of finite thickness has been studied in [10,11] with both sliding and rolling friction. We remark that in all the articles we have found in the literature for the disk motion with friction, the initial conditions are such that the disk always falls over its base. In contrast, we deal with a problem whose initial conditions ensure that the cylinder always falls over one of its edges. Related with this problem is the study of the motion for a conical frustum with sliding friction but without rolling friction in [17]. Also, an interesting study about the rocking by rolling of a cylindrical container has been made in [18].

In our study of the cylinder motion we have considered the sliding and rolling cases, both with dissipation, although without air viscous dissipation. To simulate the pivoting dissipation we also introduce a friction torque which realizes that we have not a contact point but a small contact region. To obtain more accurate equations for the case of an elongated cylinder, we have also introduced the airflow effect using the Kutta–Joukowski lift theorem. On the other hand, our work can be easily generalized to the case of a rotationally symmetric body whose contact point with the horizontal plane is in a circle. These mechanical problems are present in many real situations.

Section 2 is divided into three parts. In the first part, we set the notation and establish the general equations of the motion. Then in the following parts, we specialize to the equations for the sliding and rolling cases. In the sliding case, we find six differential equations giving the evolution of the two components of the velocity of the contact point, the three components of the angular velocity, and the inclination angle. In the rolling case, the velocity of the contact point is null, thus we have to determine only four differential equations giving the evolution of the three components of the angular velocity, and of the inclination angle. The details of the calculations are given in the Appendices.

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Video S1. Motion that acquires an AA battery after a snap of a finger (slow motion version).

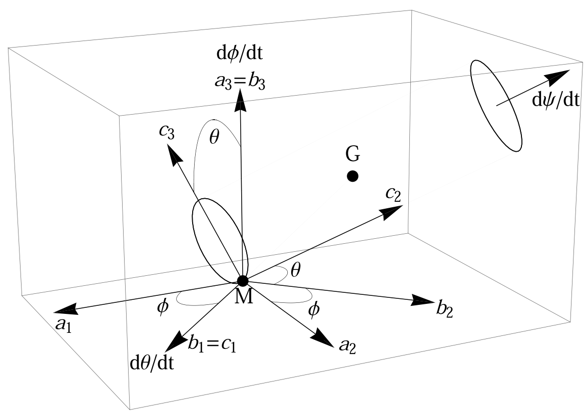

Fig. 1. Euler angles (ϕ, θ, ψ) and the coordinate axes used to define the orientation of the cylinder. Below, the AA battery for which we have made the numerical simulation, before and after we give it a fast spin with a snap of a finger.

Section 3 is also divided into three parts. In the first part, we show the results of the numerical integration of the previously obtained differential equations. In order to simulate the rolling and sliding motion of the cylinder, it is assumed that the change in the motion from rolling to sliding occurs when static friction is less than kinetic friction, whereas the change from sliding to rolling occurs when the velocity of the contact point is less than a small fixed value. We consider initial conditions that match with our experiment (initial inclination angle $\simeq 0$ and initial spinning velocity not too high). We compare the behavior of the cylinder with and without airflow effect and analyze all the relevant dynamical variables of the motion. In particular, our study shows that after a transitory state in which the cylinder inclination angle rises with large oscillations, it goes to a quasi-stationary state during which the rotation and precessional angular velocity remains very high whereas the inclination angle decreases very slowly. We have concluded that the results of the simulations are in good agreement qualitatively with the observed real motion. In the second part of the section we show that, in general, there exits three regions for the inclination angle, for which the cylinder has different qualitative dynamical behavior. In the last part of the section, we give some simple qualitative arguments about the dynamics of the rolling and sliding cylinder.

2. Equations of motion

2.1. General equations of motion

Consider a cylinder that rolls or slides, under the action of gravity, with a single point of contact with the horizontal plane. Let m be the mass, R the radius, and L the length of the cylinder.

The position of the cylinder is given by the coordinates of its center of mass G with respect to an inertial reference frame and the associated Euler angles. Following Goldstein [19], let us denote by (ϕ, θ, ψ) the Euler angles. We will consider three vectorial bases $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ (see Fig. 1) with the following properties:

(a) $\mathcal{A} = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ denotes a right oriented orthonormal basis, with $\mathbf{a}_3$ in the vertical direction, so that the gravity field is $\mathbf{g} = -g\mathbf{a}_3$.
(b) $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is the basis obtained by rotating $\mathcal{A}$ around $\mathbf{a}_3$ an angle ϕ.