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On the secondary resonance of a MEMS resonator: A conceptual study based on shooting and perturbation methods

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ABSTRACT

Nonlinear dynamics of a clamped–clamped capacitive micro-beam resonator subjected to subharmonic excitation of order one-half is studied. The micro-beam resonator is sandwiched with two piezoelectric layers throughout the length, and as a result of piezoelectric actuation a tensile/compressive axial load is induced along the length which is used as a frequency tuning tool. The resonator is subjected to a combination of a bias DC and harmonic AC electrostatic actuations. In order to determine the frequency response subharmonic resonance condition, both perturbation and shooting methods are applied. The stability of the periodic solutions and the bifurcations types are also studied. It is shown that the application of perturbation method imposes some limitations on the order of magnitudes of the terms in the differential equation of the motion; as a result out of the domain where the ordering assumption of the perturbation solution does not hold, some periodic solutions as well as some vital bifurcation points are missed. It is shown that on the frequency domain, the resonator exhibits both softening and hardening behaviors whereas this is not predicted by the perturbation scheme. The effect of DC and AC actuation voltages on the qualitative response of the system is determined. It is shown that based on the polarity of the piezoelectric actuation, the frequency response curves can be shifted both in forward and backward directions which can be used in the design of novel RF MEMS filters/sensors.

1. Introduction

It is uncertain when Micro-electro-mechanical Systems (MEMS) technology was born [1]. From the point of view of the fabrication process, the steps developed together with the semiconductor technologies started in the 1950s [1]. However, MEMS have been developed since the 1970s for pressure and temperature sensors, accelerometers and other sensor devices. MEMS switches for low-frequency applications have also been established in the early 1980s but remained a laboratory curiosity for a long time [2]. But in 1991, under the support of DARPA (Defense Advanced Research Projects Agency), Larry Larson at the Hughes Research Labs industrialized the first MEMS switch that was specifically designed for microwave applications [3]. There are a lot of challenges that face MEMS engineers and researchers when trying to model and simulate the mechanical and electrical behavior of MEMS devices. The most challenging part is nonlinearity. In MEMS, nonlinearities can be strong and dominant [4]. So, neglecting them can lead to specious predictions. There are numerous sources of nonlinearities in MEMS, which are due to forcing, damping, and stiffness [4]. The most common kind of forcing in MEMS is electrostatic force. The electrostatic force can be a combination of a DC voltage [5–8] and a harmonic AC voltage [9–14]. Another important source of nonlinearity in MEMS is the stiffness which is due to axial stretching of the clamped–clamped micro-beam under large-amplitude deflection [15]. The presence of these sources of nonlinearities leads to the introduction of various strange dynamic behaviors. The most interesting behavior is the generation of primary and secondary resonances. Primary resonance can occur when the excitation frequency of a micro-beam is near to one of its natural frequencies [16]. Primary resonance was studied experimentally [17–22] and analytically [12,14,23–29]. Secondary resonance can occur when the micro-beam is excited by a harmonic force of a frequency that is away from one of its natural frequencies [4]. If the excitation frequency of the system is two times the natural frequency, it is called subharmonic resonance of order one-half. Quadratic nonlinearity produces superharmonic resonance of order two if the system is excited near $\omega_n/2$.

In 2003 Abdel-Rahman and Nayfeh [30,31] studied the response of a micro-beam-based resonant sensor to

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superharmonic and subharmonic electric actuations using the method of multiple scales. They presented frequency response and force response curves and investigated the solution corresponding to superharmonic and subharmonic excitations. The results provided an analytical tool to predict the response of a micro-beam to superharmonic and subharmonic excitations, specifically the locations of sudden jumps. Nayfeh and Younis in 2005 [11] considered a clamped–clamped micro-beam and studied the frequency response of superharmonic and subharmonic excitations separately. In the case of superharmonic resonance, they studied the effect of varying the DC bias, the damping and the AC excitation amplitude on the frequency response curves. On the other hand, in the case of subharmonic excitation, they presented that, once the subharmonic resonance is triggered, regardless of the magnitude of the AC forcing, all frequency response curves reach pull-in. They also concluded that the quality factor has a limited effect on the frequency response in subharmonic excitation of order one-half. In 2009 Najar et al. used differential quadrature method (DQM) and finite difference method (FDM) to study the dynamic behavior of an electrostatically actuated micro-beam. They investigated the limit cycle solutions using DQM-FDM technique and the stability of these solutions was ascertained using Floquet theory. The method was applied for large excitation amplitudes and large quality factors for primary and secondary resonances in case of hardening type and softening type behaviors. They showed that the combined DQM–FDM method improves convergence of the dynamic solutions. Alsaleem et al. [25] explored the nonlinear resonances of MEMS resonators theoretically and confirmed their results experimentally. They verified that the primary and subharmonic resonances show large deflection with softening behavior while superharmonic resonance shows almost linear behavior with the weakest signal. They found that the sharpness of subharmonic resonance was amplified significantly when increasing the AC voltage. In 2014 Younesian et al. [32] studied the nonlinear dynamics of a two-side electrostatically actuated capacitive micro-beam. Higher-order nonlinear terms in the equation of motion and the nonlinear elastic terms, as well as, the inertial terms were taken into consideration. The nonlinear governing equation of the motion was derived and the method of multiple scales was applied for solving it.

The nonlinear dynamics of micro-resonators in the vicinity of both primary and secondary resonances have numerously been investigated [8,11,12,14,23,30–32]. To capture the periodic orbits multiple time scales of perturbation method [6,14,32] and Shooting technique [14,24] have been used as two worthy tools in the determination of the frequency response curves. It is shown that based on the polarity of the piezoelectric actuation, the frequency response curves can be shifted both in forward and backward directions which can be used in the design of novel RF MEMS filters [33,34]. Due to the sharp passband-to-stopband transitions, subharmonic resonance of order one-half is the best candidate for designing RF MEMS filters [11]. In the vicinity of the subharmonic resonance, most of the studies have applied multiple scales method to determine the frequency response curves which limits the amplitude and the frequency of the harmonic excitation as well as the damping coefficient to be at definite orders of the small book keeping parameter ε. In this paper, we focus on the nonlinear dynamics of a clamped-clamped micro-beam resonator in the vicinity of subharmonic resonance of order one-half. The periodic orbits and the corresponding frequency response curves are determined both using multiple scales of perturbation technique and shooting methods. It is shown that in order to apply perturbation method, the amplitude of the motion, the amplitude and frequency of the harmonic excitation, as well as the damping coefficient need to remain at a definite order of magnitude, so that the assumptions corresponding to the balancing of terms hold. It is demonstrated that due to these limitations the perturbation method in not capable of capturing some periodic orbits and cyclic fold bifurcations on the frequency response curves; these periodic orbits are captured by means of shooting method and the stability of the periodic orbits is determined using Floquet theory.

2. Modeling

The proposed model is a clamped-clamped capacitive micro-beam of length l, thickness h, and width a. The micro-beam is sandwiched with two piezoelectric layers throughout the entire length. The resonator is subjected to a stationary electrode located on the bottom of the micro-beam with initial gap denoted by g. As depicted in Fig. 1, through the electrode a combination of DC and an AC harmonic voltage with amplitude V_{AC} and frequency Ω is applied. Piezoelectric layers are actuated by applying the piezoelectric voltage (V_p) to each layer.

Fig. 2 depicts the configuration of an element of the beam before and after loading.

The length of the element before and after deflection are denoted by dx and $d\tilde{x}$, respectively. $w(x,t)$ and $u(x,t)$, refer to the lateral and longitudinal deflections of the micro-beam. Due to the immovable edges, DC piezoelectric actuation leads in the generation of mechanical stress and accordingly an axial load $(F_p(t))$ along the length of the micro-beam. $f_{es}(w,t)$ is the nonlinear displacement dependent electrostatic force.

Two material points p and p' are assumed on either sides of the element. Undergoing deformation, the position of these points in the deformed configuration become ($\tilde{x}=x+u,\ \tilde{z}=w$) and ($\tilde{x}+d\tilde{x}$, $w+d\tilde{z}$) respectively. The length of the element after deformation is approximated as:

$$ds=\sqrt{d\tilde{x}^2+d\tilde{z}^2}=\sqrt{(1+u')^2+w'^2}dx \quad (1)$$

where prime denotes differentiation with respect to x. Applying Newton's second law of motion, expanding θ in Taylor series and neglecting the higher order terms, the equation of motion in the x

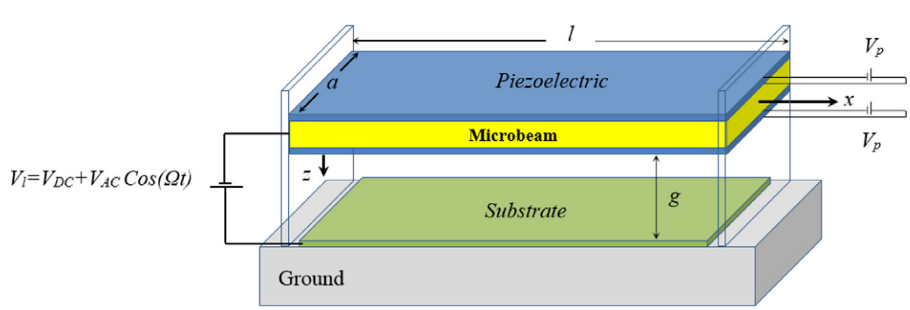

Fig. 1. A schematic of a MEMS resonator and the applied voltages.