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Effects of an impermeable wall in dissipative dynamics of saturated porous media

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ABSTRACT

A phase transition model for porous media in consolidation is studied. The model is able to describe the phenomenon of fluid-segregation during the consolidation process, i.e., the coexistence of two phases differing on fluid content inside the porous medium under static load. Considering pure Darcy dissipation, the dynamics is described by a Cahn–Hilliard-like system of partial differential equations (PDE). The goal is to study the dynamics of the formation of stationary fluid-rich bubbles. The evolution of the strain and fluid density profiles of the porous medium is analysed in two physical situations: fluid free to flow through the boundaries of the medium and fluid flow prevented at one of the two boundaries. Moreover, an analytic result on the position of the interface between the two phases is provided.

1. Introduction

In the context of soil consolidation, one can consider the problem of fluid-segregation. Terzaghi's and Biot's theories [1–3] admit only one stationary state. On the other hand the so-called Mandel–Cryer effect, see [4,5], is capable to predict fluid-segregation for special geometries of the porous material and for short-time only.

In the recent literature an extension of the classical Biot theory has been formulated via a non-linear poromechanical model within the framework of second gradient theories [6–9]. This model is able to describe the occurrence of a second stationary state richer in fluid. Thus, fluid-segregation is described by considering a kind of phase transition.

The model in [6–9] is based on a quartic energy potential, depending on the strain ε and the variation of the fluid mass density m. This potential is an extension to that of Biot [2] and, beside the new fluid-rich phase, it provides the same Biot's standard phase. The dissipation process which characterizes the dynamics of consolidation is described considering a pure Darcy dissipation rather than a pure Stokes dissipation. It is possible to prove that Darcy dissipation implies the behaviour of the porous system to be governed by a kind of Cahn–Hilliard set of partial differential equations (PDE), whilst pure Stokes dissipation yields a kind of Allen–Cahn set of equations [9]. The effect of the external world on the consolidating porous medium can be coded into the boundary conditions of the PDE problem describing the evolution of the system. The main mathematical difference between the two PDE cases is that the Allen–Cahn-like equation is a second-order PDE, while the Cahn–Hilliard equation a fourth-order one. Thus, in the latter case two additional boundary conditions have to be prescribed.

Here we focus on the effects of an impermeable wall on the consolidation process. The main appropriate equation in this context is the Cahn–Hilliard one; in fact in this case we can prescribe boundary conditions on the chemical-potential or on its derivative (seepage velocity), while in the Allen–Cahn case, being this choice impossible, the chemical potential is fixed to be zero at the boundaries, thus letting the fluid be free to flow.

In [10], as an extension to the model [6–9], the authors introduced an impermeable wall in one of the boundaries; in order to obtain the fluid-segregation inside the porous medium, the fluid-poor and the fluid-rich phases at the boundaries of the medium itself were prescribed via Dirichlet boundary conditions. The numerical analysis performed there allowed to describe how the segregation occurs: independently of the presence of the wall, the dynamics is divided into two steps, the formation of the interfaces and the motion towards the stationary profile.

In the present work we discuss the effects of the impermeable wall by choosing a set of Neumann homogeneous boundary conditions. We find that in the case of impermeable wall the dynamics ends with the formation of an interface between the fluid-poor and the fluid-rich phases, while the dynamics without the wall reaches the standard homogeneous Biot phase [2] mentioned above. Regarding the study of the stationary problem, we are able to prove analytically that the position of the interface in this case

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of Neumann homogeneous boundary conditions is the same as the Dirichlet case, a result firstly discussed in [11]. For a detailed discussion regarding the choice of these boundary conditions we refer to Section 2.5.

The paper is organized as follows. In Section 2 we summarize the model in [6–9] and we discuss the physical meaning of the boundary conditions under consideration. In Section 3 we discuss numerical simulations of the stationary problem and we prove an analytic result regarding the position of the interface between the two phases. In Section 4 we discuss the dynamical problem, pointing out the role of the impermeable wall in one of the boundaries of the porous material. Finally, in Section 5 we summarize the main results of this work.

2. The model

In this section we summarize the model in [6–9] with the extension made in [10]. Moreover we present a discussion on the meaning of the boundary conditions taken into account and we apply the model to a special choice of fourth-order overall potential energy, see Section 2.4.

2.1. Equations of motion

We summarize the one dimensional poromechanical model introduced in [9], where the authors have derived the equations of motion by using a variational approach much similar to that developed in [12]; for more details we refer to [9].

Let $B_s:=[\ell_1, \ell_2] \subset \mathbb{R}$, with $\ell_1, \ell_2 \in \mathbb{R}$, and $B_f:=\mathbb{R}$ be the *reference* configurations for the solid and fluid components, see [13]. The *solid placement* $\chi_s: B_s \times \mathbb{R} \to \mathbb{R}$ is a C^2 function such that the map $\chi_s(\cdot, t)$, associating to each $X_s \in B_s$ the position occupied at time t by the particle labelled by X_s in the reference configuration B_s, is a C^2-diffeomorphism. The *fluid placement* map $\chi_f: B_f \times \mathbb{R} \to \mathbb{R}$ is defined analogously. The *current configuration* $B_t:=\chi_s(B_s, t)$ at time t is the set of positions of the superposed solid and fluid particles.

Consider the C^2 function $\phi: B_s \times \mathbb{R} \to B_f$ such that $\phi(X_s, t)$ is the fluid particle that at time t occupies the same position of the solid particle X_s; assume, also, that $\phi(\cdot, t)$ is a C^2-diffeomorphism mapping univocally a solid particle into a fluid one. The three fields χ_s, χ_f, and ϕ are not at all independent.[1]

The *Lagrangian velocities* are the two maps $u_\alpha: B_\alpha \times \mathbb{R} \to \mathbb{R}$ defined by setting $u_\alpha(X_\alpha, t):=\partial\chi_\alpha/\partial t$ for any $X_\alpha \in B_\alpha$, where $\alpha = s, f$. We also consider the *Eulerian velocities* $v_\alpha: B_t \times \mathbb{R} \to \mathbb{R}$ associating with each point $x \in B_t$ and for each time $t \in \mathbb{R}$ the velocities of the solid and fluid particles occupying the place x at time t; more precisely we set $v_\alpha(x, t):=u_\alpha(\chi_\alpha^{-1}(x, t), t)$.

Since the reference configuration B_s of the solid component is known a priori, we express the dynamical observables in terms of the fields χ_s and ϕ which are defined on B_s.

Assume that the effect of the internal forces exchanged by the solid and fluid particles and that of the conservative external fields can be described via a potential energy density Φ depending on the kinematic fields χ_s and ϕ only through the strain and a properly normalized fluid mass density [9], i.e.,

$$\varepsilon(X_s, t):=[(\chi_s'(X_s, t))^2 - 1]/2 \quad \text{and} \quad m_f(X_s, t) := \varrho_{0,f}(\phi(X_s, t))\phi'(X_s, t) \tag{1}$$

where $\varrho_{0,f}: B_f \to \mathbb{R}$ is a fluid reference *density*. In other words, we assume that the potential energy density Φ is a function of the fields m_f and ε and on their space derivative m_f' and ε'.

[1] Indeed, by definition, we immediately have that $\chi_f(\phi(X_s, t), t) = \chi_s(X_s, t)$ for any $X_s \in B_s$ and $t \in \mathbb{R}$.

By a standard variational computation, see [9, equation (24)], one gets the equation of motion. In this framework, we are interested in the geometrically linearized version of such equations: we assume $\varrho_{0,f}$ to be constant and introduce the *displacement fields* $u(X_s, t)$ and $w(X_s, t)$ by setting

$$\chi_s(X_s, t) = X_s + u(X_s, t) \quad \text{and} \quad \phi(X_s, t) = X_s + w(X_s, t) \tag{2}$$

for any $X_s \in B_s$ and $t \in \mathbb{R}$. We then assume that u and w are small, together with their space and time derivatives, and write

$$m_f = \varrho_{0,f}(1 + w'), \quad m:=m_f - \varrho_{0,f} = \varrho_{0,f}w', \quad \varepsilon \approx u', \tag{3}$$

where $\approx$ means that all the terms of order larger than one have been neglected.

We have introduced above the field m. In the following we shall imagine Φ as a function of m, ε and $m'\varepsilon'$ and the equations of motion and the boundary conditions will be written in terms of these fields. We get the equations of motion [10]

$$\frac{\partial\Phi}{\partial\varepsilon} - \left(\frac{\partial\Phi}{\partial\varepsilon'}\right)' = 0 \quad \text{and} \quad \varrho_{0,f}^2\left[\frac{\partial\Phi}{\partial m} - \left(\frac{\partial\Phi}{\partial m'}\right)'\right]'' = D\dot{m}, \tag{4}$$

and the associated boundary conditions that are compatible with the choices of Dirichlet and Neumann boundary conditions:

$$\left\{\left(\frac{\partial\Phi}{\partial\varepsilon'}\delta\varepsilon + \frac{\partial\Phi}{\partial m'}\delta m\right) + \left[\left(\frac{\partial\Phi}{\partial m} - \left(\frac{\partial\Phi}{\partial m'}\right)'\right)\varrho_{0,f}\right]\delta w\right\}_{\ell_1}^{\ell_2} = 0 \tag{5}$$

Recalling that in our approximation $m = \varrho_{0,f}w'$, see the second among Eqs. (3), we have that Eqs. (4) are evolution equations for the fields m and ε.

The second between the equations of motion (4), thanks to a suitable choice of Φ, see Section 2.4, will become a Cahn–Hilliard-like equation for the field m with driving field still depending parametrically on ε [9].

2.2. The zero chemical potential problem

A set of boundary conditions implying that (5) are satisfied is

$$\left(\frac{\partial\Phi}{\partial\varepsilon'}\delta\varepsilon + \frac{\partial\Phi}{\partial m'}\delta m\right)_{\ell_1,\ell_2} = \left[\frac{\partial\Phi}{\partial m} - \left(\frac{\partial\Phi}{\partial m'}\right)'\right]_{\ell_1,\ell_2} = 0 \tag{6}$$

where the notation above means that the functions in brackets are evaluated both in ℓ_1 and ℓ_2. With this choice it is possible to fix the boundary conditions directly on fields m and ε (and derivatives).

The first equation (6) is the additional boundary condition due to the presence of the gradient terms in the potential energy density Φ. This equation specifies essential boundary conditions on the derivatives of the displacement fields or natural boundary conditions on the so-called double forces, see [14] and the next Section 2.5. The generalized essential boundary conditions can be read as a prescription on the derivative of the independent fields χ_s and ϕ, see Eq. (1); whilst the extended natural boundary conditions prescribe, on one hand, the additional forces which the solid continuum is able to balance at the boundary and, on the other, the wetting properties of the fluid which fills the pores [15].

The second equation (6) provides natural boundary conditions prescribing the chemical potential of the fluid, so that the fluid is free to flow through both the two boundaries. For a more detailed discussion on this boundary condition see [10,16].

Finally, we call the *zero chemical potential* problem the PDE problem given by (4) and (6).

2.3. The one-side impermeable problem

A very interesting situation in applications is the one in which