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Non-isothermal flows of liquid metals and melts: Qualitative features, asymptotic models, problems, exact and approximate solutions

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ABSTRACT

We analyze experimental thermophysical properties of liquid metals and alloy melts and show that the kinematic viscosity is essentially dependent on temperature, whereas the density, thermal conductivity, and specific heat capacity are only weakly dependent on temperature. Based on this fact, we formulate a mathematical model for non-isothermal laminar flows of liquid metals and melts with variable viscosity. We derive asymptotic equations of motion for low Prandtl numbers (liquid metals are characterized by $\mathrm{Pr} \simeq 10^{-2} \ll 1$) and different Reynolds numbers and obtain a number of exact and approximate analytical solutions expressible in elementary functions or representable in closed form. We look at a few specific fluid and thermodynamic problems and show that the dependence of viscosity on temperature significantly affects the drag coefficient in non-isothermal flows as compared to isothermal flows. We outline a few semi-empirical approximations of $\nu(T)$ and show that the power-law formula $\nu = \nu_0(T_0/T)^k$ provides a very good accuracy for several liquid metals (including sodium and mercury). The asymptotic models, equations and formulas presented in the paper can be used to state and solve new non-isothermal hydrodynamic problems for liquid metal and melt flows.

1. Introduction

1.1. Classical mathematical models of fluid dynamics. Exact solutions

The classical mathematical models describing laminar flows of viscous incompressible fluids are based on the Navier–Stokes equations and boundary layer equations. These equations as well as various statements of problems and their solutions are discussed in numerous studies, for example, [1–10].

Exact solutions to the Navier–Stokes, boundary layer, and related equations contribute to better understanding of qualitative features of steady and unsteady fluid flows at large Reynolds numbers; these features include stability, non-uniqueness, spatial localization, blow-up regimes, and others.

Exact solutions with significant functional arbitrariness are of particular interest because they may be used as test problems to ensure efficient estimates of the domain of applicability and accuracy of numeric, asymptotic, and approximate analytical methods for solving suitable non-linear hydrodynamic-type PDEs as well as certain model problems.

Steady and unsteady solutions to two- and three-dimensional Navier–Stokes equations can be found in [2,5,7,10–21,23,22]. For models and exact solutions to hyperbolic and differential–difference Navier–Stokes equations, see [24–27].

For exact solutions to steady and unsteady plane boundary-layer equations, see [5,7,20,21,23,28–41]. The studies [21,38,41–46] present some exact solutions and reductions of axisymmetric boundary-layer equations.

1.2. Convective heat and mass transfer equations. Analytical solutions

The classical mathematical models describing convective heat and mass transfer in a fluid flow are based on convective heat and mass transfer equations. These equations as well as various

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statements of problems and solution methods are detailed, for example, in [5,7,47–62].

Most studies assume that fluid viscosity is independent of temperature. As a result, the overwhelming majority of convective heat and mass transfer problems are characterized by passive heat and mass transfer equations,[1] in the sense that these equations do not affect the fluid velocity field, while being dependent on this velocity field due to convective terms of the form $(\mathbf{u}\cdot\nabla)T$. Consequently, the solution of these equations splits into two consecutive stages: (i) solving the fluid dynamic component of the problem (to determine the velocity field) followed by (ii) solving the thermal and diffusion components of the problem with the velocity field already known. Diffusion problems for ordinary liquids like water and viscous liquids like glycerol and various oils are characterized by large Schmidt numbers ($\mathrm{Sc}\simeq 10^3$–10^6). It follows that, starting from fairly small Reynolds numbers, $\mathrm{Re}\gtrsim 0.1$, the corresponding diffusion Péclet numbers, $\mathrm{Pe}=\mathrm{Re}\,\mathrm{Sc}$, will be large. In these cases, the diffusion boundary-layer approximation can be used; it is based on the linearization of the velocity field near the liquid–solid or liquid–liquid interface and allows one to obtain the concentration field in closed analytical form (e.g., see [51,54,58,62]).

There are a number of studies that looked at coupled hydrodynamic and heat transfer problems and considered viscous dissipation or/and the dependence of viscosity on temperature. For example, see [5,63–73]. Qualitative results of these studies are discussed below in Remarks 4 and 5.

1.3. Non-isothermal flows of liquid metals and melts

Liquid metals and alloy melts are characterized by small Prandtl numbers and essential dependence of the kinematic viscosity on temperature. These properties must be taken into account in setting and solving relevant problems.

The main objectives of the present study include:

- analyze qualitatively the physical properties of liquid metals and melts in a wide temperature range;
- assess the significant and insignificant factors affecting the modelling of non-isothermal flows of liquid metals;
- discuss semi-empirical dependences of the kinematic viscosities on temperature;
- formulate an appropriate mathematical model describing non-isothermal flows of liquid metals;
- obtain simplified asymptotic models at low Prandtl numbers and different Reynolds numbers;
- solve selected problems on the fluid and thermodynamics of liquid metals;
- construct exact and approximate solutions to the fluid and thermodynamic equations of liquid metals.

It is noteworthy that one of the important applications of liquid metals and melts is their usage as coolants at high temperatures, especially in nuclear power stations (e.g., see [74–81]).

Remark 1. Section 9 of the paper [44] discusses a model problem on a steady unidirectional flow of a liquid metal past a non-uniformly heated flat plate at large Reynolds numbers. It shows that considering the dependence of the kinematic viscosity on temperature can result in a significant change in the friction drag coefficient of the plate.

[1] Here we do not discuss free thermal convection in fluid. Neither we discuss Marangoni-type effects, thus assuming the coefficient of surface tension to be independent of temperature.

2. Thermophysical properties of liquid metals and melts

2.1. Qualitative features of liquid metals and melts

Below we briefly describe the general physical properties of liquid metals that will be required in the further statement and solution of thermal and hydrodynamic problems considering the changes in the viscosity due to temperature variations. We get the data from the thermophysical handbooks [82–84]. The following general facts are essential for further analysis:

1°: The kinematic viscosity, ν, density, ρ, and Prandtl number, $\mathrm{Pr}=\nu/a$ decrease with increasing temperature, T. The parameter $a=\lambda/(\rho c_p)$ stands for the thermal diffusivity, with λ denoting the thermal conductivity and c_p the specific heat.

2°: The changes in the density are very small compared to those in the kinematic viscosity in a wide temperature range [83,84] and so can be neglected. In particular, the kinematic viscosity of sodium changes by 52% as temperature varies from 100 to 200 °C, whereas its density changes by only 2.8%.

3°: For most liquid metals, the changes in the thermal conductivity, λ, and specific heat, c_p, are also very small as compared with those in the kinematic viscosity in a wide temperature range and so can also be neglected. In particular, the thermal conductivity and specific heat of sodium change by only 5.5% and 4.5%, respectively, across the same temperature range of 100 to 200 °C.

4°: The Prandtl number is quite small [62,84], varying within the range $5\times 10^{-3}\le \mathrm{Pr}\le 5\times 10^{-2}$.

Table 1 lists the values of relevant physical parameters of some liquid metals at different temperatures, according to the data from [84] (see also [83]).

It is apparent from the table that the following approximate relations hold:

$$\frac{\Delta\rho}{\rho}\simeq 0.1\frac{\Delta\nu}{\nu},\quad \frac{\Delta\lambda}{\lambda}\simeq 0.1\frac{\Delta\nu}{\nu},\quad \frac{\Delta c_p}{c_p}\simeq 0.1\frac{\Delta\nu}{\nu}, \tag{1}$$

where $f=f(T)$ and $\Delta f=f(T+\Delta T)-f(T)$, with $f=\rho,\lambda,c_p$.

The above properties of liquid metals and melts are key to the subsequent mathematical statements of fluid and thermodynamic problems.

2.2. Approximation of the dependence of the kinematic viscosity on temperature

In many cases, the temperature dependence of the kinematic viscosity of liquid metals and melts can be approximated by the two-parameter Arrhenius type formula (e.g., see [85,86])

$$\nu=\alpha e^{\beta/T}, \tag{2}$$

where α and β are empirical constants and T is thermodynamic temperature. The modified three-parameter formula $\nu=\alpha e^{\beta/(T+T_*)}$ provides a better approximation in a wide temperature range for different pressures; see [87]. The study [88] suggested the formula $\nu=\alpha T^{1/2}e^{\beta/T}$.

By expanding the exponential function using the method of [58], one can simplify relation (2) to obtain the following approximation, which is valid in a narrower temperature range:

$$\nu=\nu_0 e^{\gamma(T_0-T)}, \tag{3}$$

where ν_0 is the kinematic viscosity at temperature T_0 and γ is some constant (generally dependent on T_0). This formula was first suggested by Reynolds for viscous liquids like olive oil [89] (see also [90]).