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On thermo-mechanical nonlinear behaviour of shallow shells

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ABSTRACT

The structural performance of thin shells is largely dictated by their curvature and the degree of lateral restraint at the shell edges. The present study is an attempt to theoretically investigate the influence of such factors on nonlinear thermo-mechanical response of shallow shells with single and double curvatures. For the mechanical loading, a transverse load is assumed and for the thermal loading, a through-depth thermal gradient is applied on the shallow shell. Two types of boundary conditions are considered for the shallow shell, both of which constrain transverse deflections of the shell but allow rotations parallel to the shell boundaries to be free. One of the boundary conditions permits lateral translation (laterally unrestrained) and the other one does not (laterally restrained). The fundamental nonlinear equations of shallow shells are derived based on the quasi-static conditions. The validity and reliability of the proposed approach is assessed by calculating several numerical examples for shallow shells under various mechanical and thermal loads. It is found that the proposed formulation, in particular, can adequately capture the nonlinear behaviour of laterally restrained shallow shells.

1. Introduction

It is well known that shells can deliver useful load-carrying capacity by their curvatures, thereby effectively resisting the external applied loads with optimum use of material. This makes them practical and efficient structural components and of importance in the design of light weight thin-walled structures. Shell components are widely used in buildings (typically as roof structures), bridges, aerospace vehicles, ship hulls, pressure vessels and car bodies. Such components are often subjected to different thermo-mechanical loadings. Research in this area has been often focused towards developing efficient shell elements using numerical methods (e.g. see [1–9]).

However, nonlinear thermo-mechanical behaviour of shells has also been studied using adequately accurate analytical and semi-analytical methods that take into account key features and many complexities of shell problems. Woo and Meguid [10] studied the nonlinear analysis of simply supported (laterally unrestrained) shallow spherical shells with functionally graded material properties subjected to transverse mechanical loads and through-depth temperature fields. The governing equations were established based on the von Kármán theory for large transverse deflections and were solved using series solutions. It was revealed that considering thermo-mechanical coupling effects in the shell formulation can affect the nonlinear response of the shell. Nie [11] presented the nonlinear analysis of an imperfect shallow spherical shell on a Pasternak foundation subjected to uniform loads. The shell was assumed elastically restrained against rotational, transverse and in-plane displacements. The asymptotic iteration method was applied to obtain an analytical expression for the external load and the central deflection of the shell. Numerical results indicated that imperfections cause a drop in the load-bearing capacity of the shell. Heuer and Ziegler [12] studied the thermal snap-through and snap-buckling of symmetrically layered shallow shells with polygonal planforms and laterally restrained boundary conditions (BCs) using a two degrees of freedom model derived from a Ritz–Galerkin approximation.

Amabili [13] investigated the large amplitude of the response of simply supported doubly curved shallow shells with rectangular planform to static and dynamic loads. He used the Donnell and Novozhilov shell theories retaining in-plane inertia to obtain geometrically nonlinear shell responses. Duc and Van Tung [14] studied the nonlinear response of functionally graded cylindrical panels to uniform lateral pressure and uniform and through-depth temperature gradients by an analytical approach associated with a Galerkin method. Formulation was based on the classical shell theory, considering both the von Kármán–Donnell type of kinematic nonlinearity and initial geometrical imperfection. The numerical results revealed that in-plane restraint and temperature

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conditions play major roles in dictating the response of the functionally graded cylindrical panels. Recently, the nonlinear buckling behaviour of homogeneous and non-homogeneous orthotropic thin-walled truncated conical shells under axial load was presented by Sofiyev and Kuruoglu [15]. The stability and compatibility equations of the problem were derived using the large deformation theory with the von Kármán–Donnell type of kinematic nonlinearity. It was reported that for long truncated conical shells, the effect of non-homogeneity on the nonlinear axial buckling load is negligible.

Although there is no doubt that numerical methods provide greater flexibility for the analysis of shell structures when compared to analytical methods, research on improving analytical methods is also vital when computational effort (mostly in terms of analyst effort) is a concern, or when an alternative approach is required to validate and corroborate numerical results. This is particularly very useful for benchmarking finite element codes developed for thermo-mechanical simulations of shell elements [16]. Moreover, accurate mathematical models could be used to:

(i) obtain rapid solutions of realistic thermo-mechanical analyses of simple shell structures as part of Monte Carlo methods to account for uncertainty in loads and structural form;
(ii) understand the mathematical underpinnings of engineering concepts such as "thermal snap-through" in shallow shells under large displacements;
(iii) provide advanced basis functions for hybrid-type computational approaches.

Motivated by this opportunity, in the present work, a nonlinear mathematical model is developed to analyse the large deflections of shallow shells with various types of curvature under thermo-mechanical loading conditions. The shallow shell is under transverse mechanical loading while being subjected to through-depth thermal gradients. Two types of BCs are considered in the analysis: edges laterally unrestrained in translation, henceforth referred to as the "laterally unrestrained" BC; and edges restrained against translation, henceforth referred to as the "laterally restrained" BC. In both BCs, transverse deflections of the shallow shell are restrained but rotations parallel to the shell boundaries are unrestrained. The compatibility and equilibrium equations are solved for the steady-state problem using appropriate series functions. A notable feature of the proposed approach is its good performance and its relatively rapid convergence for shallow shells with the laterally restrained BC. In the case of shallow shells with the laterally unrestrained BC, this is, however, only achieved for 'extremely shallow shells' (see Fig. 1). The reason for poor agreement for this BC is the significant change in curvature which is not able to be captured by the formulation.

2. Fundamental theory

Consider a shallow shell under non-uniform through-depth thermal gradients while it is being subjected to a transverse mechanical load. In line with the assumptions of the shallow shell theory, it is assumed that the rise of the shell is relatively small in comparison to its other dimensions (see Fig. 1 which is adapted from Donnell [17] and illustrates schematically the ranges of applicability of various shell theories for modelling cylindrical shells). In the case of large deformations, the strain–displacement relations of a shallow shell with accounting for the stretching of the middle surface of the shell are expressed by

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{w}{R_x} - z\frac{\partial^2 w}{\partial x^2} \tag{1a}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{w}{R_y} - z\frac{\partial^2 w}{\partial y^2} \tag{1b}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + \frac{2w}{R_{xy}} - 2z\frac{\partial^2 w}{\partial x \partial y} \tag{1c}$$

where u, v and w are the displacements of the middle surface of the shell in x, y and z-directions, respectively, R_{xy} represents the twist radius of the middle surface of the shell, and R_x and R_y are the radii of curvature of the undeformed shell as illustrated in Fig. 2 for shallow hyperbolic paraboloidal shell (with double positive and negative curvature), shallow spherical shell (with double curvature) and shallow cylindrical shell (with single curvature). For a shell with thickness of h, elastic modulus of E, Poisson's ratio of ν, the strain components including thermal effects can be rearranged as

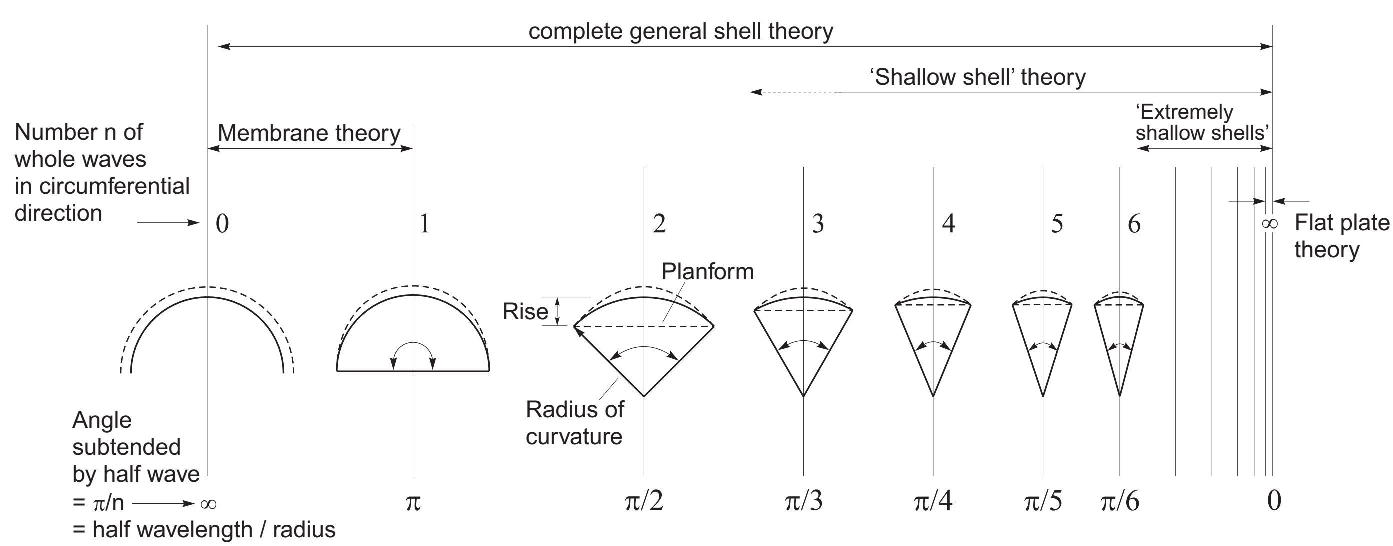

Fig. 1. Hierarchy of various shell theories as function of their applicable ranges of subtended angle. The figure is adapted from Donnell [17]. A shallow shell typically has a rise of less than one-fifth of the smallest dimension of its planform. In the case of 'extremely shallow shells', the minimum radius of curvature of the shell is more than two times larger than its maximum planform dimension [18].