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Steady state dynamics of a non-linear beam coupled to a non-linear energy sink

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ABSTRACT

A non-linear, simply supported beam under harmonic excitation coupled to a non-linear energy sink (NES) is considered here. The NES has a non-linear stiffness of order three. Steady state dynamic of the beam is investigated by two different theories of Euler–Bernoulli and Timoshenko. Complex averaging method combined with arc-length continuation is used to achieve an approximate solution for the steady state vibrations of the system based on 1:1 resonance condition. In order to design an optimized NES for the purpose of reducing the vibration amplitude of the beam, the effect of NES parameters on the amplitude of the primary system is investigated by varying the parameters, individually. The results demonstrated a significant reduction in the vibration amplitude of the original system. By illustrating the frequency spectrum, other harmonic components are detected and the steady state dynamic of the non-linear primary system is computed including the higher harmonics. Non-linear dynamic studies such as bifurcation analysis and Poincare' sections are also applied in order to study the effect of NES on the vibration behavior of the beam, in a more accurate manner. Numerical simulations confirm the accuracy of the approximate solutions. Robustness of the NES against changes in the amplitude of excitation is also investigated. Also the performance of NES is compared with linear vibration absorber.

1. Introduction

In mechanical systems, the amount of undesired vibrations can be reduced by vibration absorbers. The idea of reducing the dynamic motions by this device was first introduced by Frahm [1] in 1909. Later Den Hartog added another element to the device, a damper, in order to increase the effective frequency range [2].

In [3], Gendelman and his coworkers considered a system composed of weakly coupled, linear and non-linear components. Channeling of the externally imparted energy from linear oscillator to the non-linear one was evidenced in their work by numerical simulations. In another investigation, the energy pumping phenomenon was analyzed and the results showed that, it is caused by transient resonance captures. A perturbation analysis was also performed in order to identify the region responsible for this phenomenon [4]. For an asymmetric system consisting of linear and highly non-linear damped oscillators, it was evidenced that the energy transfer occurs due to the mechanism of sub-harmonic resonance which is possible because of the non-linearity of the system [5]. Investigations showed a properly designed non-linear attachment can act as a passive sink of unwanted disturbances [6]. It was shown in [7] that non-linear energy sink (NES) is capable of absorbing major portion of energy created by external forces. Resonance capture and resonance capture cascades which are responsible for single and multi-mode energy pumping were presented by numerical simulations. For a multi-degree of freedom non-linear attachment coupled to a single degree of freedom linear system, energy pumping is investigated in [8]. Results showed that the span of energy pumping increased due to the use of multiple non-linear normal modes (NNMs). Occurrence of energy pumping for lower energies was also evidenced. NNMs provide a clear understanding of a non-linear system from a linear perspective. A variety of techniques, analytical and numerical, were used in [9] to compute NNMs which stressed that a simple system may show complex behaviors. NNMs for non-linear mechanical systems such as weakly and strongly 2DOF system, a disk model and a cantilever beam were computed in [10] which made it possible for further developments. For a non-linear oscillator coupled to a linear one, it was shown in [11] that due to resonance capture the energy can be irreversibly transferred from the linear oscillator to the non-linear absorber. This non-linear energy transfer is triggered by the excitation of transient bridging orbits. For an impulsively excited linear system coupled to a non-linear energy sink, it was demonstrated that a significant amount

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of input energy can be absorbed and dissipated in this non-linear oscillator. Complicated dynamics such as non-linear beating and resonance capture was evidenced in this system as well [12].

Passive non-linear energy pumping was experimentally verified in [13]. It was also confirmed that non-linear energy pumping can occur only above a special level of input energy, and also there is an optimal value for this energy at which maximum value of absorption and dissipation occurs by NES. The incidence of transient resonance capture was studied experimentally in [14]. It was illustrated that during this phenomenon the two oscillators are in resonance which led to targeted energy transfer (TET). It was also showed that the non-time-like behavior of the phase difference between oscillators is another evidence of resonance capture. It was shown in [15] that the existence of sustained resonance capture depends on the parameters of the system. It was shown numerically and experimentally in [16] that due to the non-linearity of NES, this device can passively absorb broadband energy at both high and low frequencies. A parametric optimization was performed in [17] to investigate the efficiency of energy pumping with analytical methods. Results showed that analytical solutions can be used to find optimal parameters for NES. Experimental investigation verified the efficiency of the analytical method. The problem of choosing proper initial condition in order to reach efficient TET was investigated numerically and analytically in [18]. The approaches, which were applied in this study, yielded a complete understanding of TET, and answered the problems about robustness of energy transfer to changes in the initial conditions.

The application of TET in acoustics was introduced in [19]. In that work, a new method was provided to passively control sound in low frequency domain. Also energy transfer was evidenced in free and forced regimes by using wavelet transforms and frequency-energy plots. It was shown in [20] that using multiple NESs is a way to enhance the range of energy and the efficiency of TET. This is due to the fact that each NES was activated in turn and behaved autonomously. This paper lightened new information in the field of non-linear attachments. For an acoustic medium inside a parallelepiped cavity coupled to a thin viscoelastic membrane, the beginning of TET was analyzed, and the designed working zone for NES was identified. The formulas given in this work can estimate the numbers, places and parameters of the NES that has to be used in order to reduce the low frequency resonances [21]. It was shown in [22] that in the presence of a two-degree-of-freedom (2dof) attachment with non-linearity, energy dissipation is significantly enhanced. Multiple-scale energy exchange between a main oscillator and a non-linear energy sink is studied in [23], and different regimes which the system may encounter during energy exchange were predicted and explained. The dynamics of a Duffing oscillator coupled to a non-linear energy sink was investigated in [24]. Results showed that sub-harmonic energy pumping is the fundamental mechanism of new areas with a high capacity for energy dissipation. A non-linear model of a drill-string system coupled to NES was investigated in [25]. Results demonstrated that the flowing of targeted energy transfer to a light-weight NES can passively control the instabilities of a drill-string process.

It was shown in [26] that a non-linear coupling between oscillators allows less mass for the attachment. For a 2dof linear system coupled to NES, it was evidenced in [27] that simultaneous targeted energy transfer from both oscillators to NES is possible. For a similar system the effect of linear and non-linear absorbers was investigated in [28]. Results showed that in some cases of high amplitudes of excitations, the NES absorbs energy from both excited modes, and it is better than the linear one; but for low amplitudes of excitation, its inefficiency is notable, and also periodic regimes with high amplitudes were evidenced. These regimes can be reduced or minimized by a proper design of NES. By adding internal degree of freedom to NES, it was demonstrated in [29] that it is possible to enhance the performance of passive TET. The interesting part of this idea is that this enhancement occurs without adding another mass to the NES. For a linear subsystem coupled to NES, the possibility of chaos and transversal homoclinic orbits in the system was investigated in [30].

It was demonstrated in [31] that it is possible to apply the idea of TET and NES to seismic mitigation of structures subjected to earthquakes. The possibility of dissipating major portion of seismic energy from the primary structure was demonstrated numerically. For mitigation of seismic structural responses, a series of large-scale experimental evaluations and numerical simulations were performed in [32]. Results showed this mitigation can be achieved by a system of NES devices.

A linear cantilever beam coupled to a non-linear energy sink at its free end is considered in [33]. Results showed that non-linear stiffness mainly influence the lower frequency modes, and strongly non-linear beat phenomena is due to modal interplays of internal resonances between different modes of the beam. A method was also established to recognize these strongly non-linear modal interplays of cantilever modes. Vibration reduction of a rotating beam coupled to a non-linear energy sink is investigated in [34]. Major consideration is dedicated to investigate the effect of NES position, damping and the amplitude of the external force on vibration reduction of the system. Results showed that the best parameters for NES are obtained when strongly modulated response and weak modulated response happen at the same time. A uniform plate model of an aircraft coupled to NES is investigated in [35]. Results showed that NES can passively and sufficiently absorb energy form more than one mode of the wing. It was also illustrated that NES can be used to mitigate or even prevent aero-elastic instabilities. Parameters of a non-linear energy sink coupled to a beam with different boundary conditions are optimized in [36]. Results showed that NES activation will be delayed by increasing the first natural frequency of the beam. It was also indicated that by increasing the magnitude of the shock force, optimum value for non-linear stiffness will be achieved for lower values of stiffness. A system consisting of a NES and a piezoelectric-based vibration energy harvester attached to a free–free beam under shock excitation was considered in [37]. An optimization is performed to maximize the dissipation energy and increase harvesting energy by NES and piezoelectric element, respectively. This optimization led to 78 percent of dissipation energy. For a cantilever plate coupled to strongly non-linear attachments, TET phenomena and modal interactions are investigated in [38]. A parametric optimization indicated that the anti-nodes of the linear modes of plate are the best place to position attachments. It was also demonstrated that TET decrease by decreasing the dissipation elements of the non-linear attachments. Comparison between NES and tuned mass damper (TMD) showed improved robustness for NES and its effective role in broadband vibration absorption. It was also concluded that TMD is not suitable for vibration mitigation of systems under multiple successive shocks.

The effect of NES on the steady state dynamics of a weakly coupled system was investigated theoretically and experimentally in [39]. Results showed that NES is capable of absorbing energy over a broad frequency range. In contrast to the previous idea, for the same system, Malatkar and Nayfeh showed that NES results in an increase in the vibration amplitude of the linear subsystems, especially for lower values of damping. Also the occurrence of energy transfer was ruled out for systems with weak damping and non-linearity [40]. By comparing the steady state response of the original primary system to the case without NES, noticeable