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A numerical study for three-dimensional viscoelastic flow inspired by non-linear radiative heat flux

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ABSTRACT

Present article examines the three-dimensional flow of upper-convected Maxwell (UCM) fluid over a radiative bi-directional stretching surface. Novel non-linear Rosseland formula for thermal radiation is utilized in the formulation of energy equation. The conventional transformations lead to a strongly non-linear differential system which is treated numerically through Runge–Kutta integration procedure together with the shooting approach. We found that heat transfer rate from the sheet has inverse as well as non-linear relationship with wall to ambient temperature ratio. Moreover an increase in viscoelastic fluid parameter (Deborah number) corresponds to a decrease in the fluid velocity and the boundary layer thickness.

1. Introduction

Wang [1] was probably the first to explore the three-dimensional flow induced by a bi-directionally stretching surface. He successfully developed a self-similar solution of the three-dimensional Navier–Stokes equations. Ariel [2] found the series solution for Wang's problem by using the homotopy perturbation method (HPM). Liu and Andersson [3] investigated the heat transfer over a bidirectional stretching sheet with variable thermal conditions. Homotopy analysis method (HAM) based analytical study of three-dimensional flow and heat transfer above an implusively stretching surface was presented by Xu et al. [4]. Sajid et al. [5] also derived homotopy solutions for three-dimensional flow of Walters B liquid over a linearly stretching surface. Time-dependent flow of Maxwell fluid bounded by an unsteady stretching sheet was addressed by Awais et al. [6]. Khan et al. [7] discussed the three-dimensional flow and heat transfer caused by a stretching surface subject to general power-law surface velocity and temperature distribution. Xu and Pop [8] discussed the impact of bio-convection on nanofluid flow through a vertical channel containing gyrotactic microorganisms by optimal homotopy analysis approach. Very recently, three-dimensional flow of nanofluids induced by a non-linearly stretching sheet was investigated by Khan et al. [9]. Revolving flow over a stretching disk with heat transfer was numerically examined by Turkyilmazoglu [10].

The phenomenon of radiative heat transfer has relevance in numerous industrial applications including power generation, combustion applications, nuclear reactor cooling etc. Raptis [11] investigated a boundary layer flow problem considering thermal radiation effect. He linearized the Rosseland formula for thermal radiation by assuming small temperature differences within the flow. In a recent article, Magyari and Pantokratoras [12] showed that linear radiation heat transfer problem reduces to a simple re-scaling of Prandtl number by a factor containing the radiation parameter. Keeping this in view, Rahman and El-tayeb [13] considered the radiation effects on the flow of an electrically conducting nanofluid by using the exact Rosseland formula. Pantokratoras and Fang [14] investigated the Sakiadis and Blasius flow problems considering the non-linear radiation. Recent studies pertaining to the non-linear radiation heat transfer in the boundary layer flows can be found in the Refs. [15–19].

The purpose of current work is to investigate the non-linear radiative heat transfer in the three-dimensional flow of UCM fluid bounded by a bi-directional stretching surface. Upper-convected Maxwell (UCM) fluid is a subclass of rate type fluids which is important in describing the influence of fluid relaxation time. It has gained special attention of the researchers in the past due to its simplicity. Recently, various papers involving the flow analysis of Maxwell fluid have appeared (see for instance [20–26]). It will

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be seen later that consideration of non-linear radiative heat flux produces a strongly non-linear but interesting energy equation for the temperature field. Shooting method together with fifth-order Runge–Kutta integration and Newton method is employed for the development of numerical solution. Computational results for both viscous and Maxwell fluids are presented in a tabular form. Graphical results for the velocity and temperature distributions are also presented and analyzed.

2. Basic equations

Consider three-dimensional flow of upper-convected Maxwell (UCM) fluid induced by a stretching surface occupying the xy-plane (see Fig. 1). The velocities of the sheet along the x- and y-directions are assumed to be $U_w(x)=ax$ and $V_w(y)=by$ respectively in which $a,b>0$ are constants. Let T_w be the constant temperature at the sheet whereas T_∞ denotes the fluid temperature outside the thermal boundary layer. The equations governing the three-dimensional flow of UCM fluid with radiative heat transfer are expressed below (see Liu and Andersson [3] and Awais et al. [6] for details):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+\lambda_1\left(\begin{array}{c} u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+w^2\frac{\partial^2 u}{\partial z^2} \\ +2uv\frac{\partial^2 u}{\partial x\partial y}+2uw\frac{\partial^2 u}{\partial x\partial z}+2vw\frac{\partial^2 u}{\partial y\partial z}\end{array}\right)=\nu\frac{\partial^2 u}{\partial z^2}, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+\lambda_1\left(\begin{array}{c} u^2\frac{\partial^2 v}{\partial x^2}+v^2\frac{\partial^2 v}{\partial y^2}+w^2\frac{\partial^2 v}{\partial z^2} \\ +2uv\frac{\partial^2 v}{\partial x\partial y}+2uw\frac{\partial^2 v}{\partial x\partial z}+2vw\frac{\partial^2 v}{\partial y\partial z}\end{array}\right)=\nu\frac{\partial^2 v}{\partial z^2}, \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial z}, \tag{4}$$

where u, v and w are the velocity components along the x-, y- and z-directions respectively, λ_1 is the fluid relaxation time, ν is the kinematic viscosity, α is thermal diffusivity of the fluid, ρ is the fluid density, C_p is the specific heat, $q_r=-(4\sigma^*/3k^*)\partial T^4/\partial z$ is the Rosseland radiative heat flux in which σ^* is the Stefan–Boltzman constant and k^* is the mean absorption coefficient respectively. The boundary conditions in the present problem are:

$$\begin{array}{ll} u=U_w=ax, v=V_w=by, w=0,\ T=T_w & \text{at } z=0, \\ u,v\to 0, T\to T_\infty & \text{as } z\to\infty. \end{array} \tag{5}$$

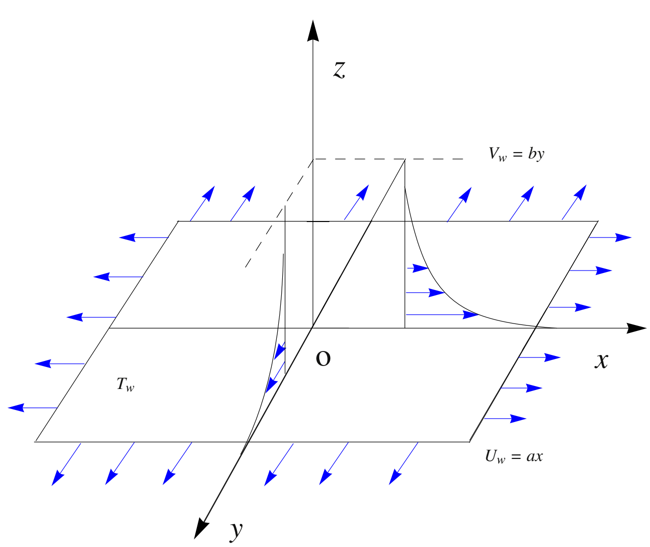

Fig. 1. A schematic diagram showing the development of boundary layer.

We now introduce the following non-dimensional quantities

$$\eta=z\sqrt{\frac{a}{\nu}}, u=ax\left(\frac{df}{d\eta}\right), v=ay\left(\frac{dg}{d\eta}\right), w=\nu\sqrt{\nu a}(f+g), \theta=\frac{T-T_\infty}{T_w-T_\infty}. \tag{6}$$

In view of the above variables, the continuity Eq. (1) is automatically satisfied and the Eqs. (2)–(5) become

$$\frac{d^3f}{d\eta^3}-\left(\frac{df}{d\eta}\right)^2+(f+g)\frac{d^2f}{d\eta^2}+K\left[2(f+g)\frac{df}{d\eta}\frac{d^2f}{d\eta^2}-(f+g)^2\frac{d^3f}{d\eta^3}\right]=0, \tag{7}$$

$$\frac{d^3g}{d\eta^3}-\left(\frac{dg}{d\eta}\right)^2+(f+g)\frac{d^2g}{d\eta^2}+K\left[2(f+g)\frac{dg}{d\eta}\frac{d^2g}{d\eta^2}-(f+g)^2\frac{d^3g}{d\eta^3}\right]=0, \tag{8}$$

$$\frac{1}{\Pr}\frac{d}{d\eta}\left[\left(1+Rd(1+(\theta_w-1)\theta)^3\right)\frac{d\theta}{d\eta}\right]+(f+g)\frac{d\theta}{d\eta}=0, \tag{9}$$

$$\begin{array}{ll} f=g=0,\ \frac{df}{d\eta}=1,\ \frac{dg}{d\eta}=c, \theta=1 & \text{at} \quad \eta=0, \\ \frac{df}{d\eta}\to 0,\ \frac{dg}{d\eta}\to 0,\ \theta\to 0 & \text{as} \quad \eta\to\infty. \end{array} \tag{10}$$

In the above equations $K=\lambda_1 a$ is the Deborah number, $c=b/a$ is the stretching rates ratio, $\theta_w=T_w/T_\infty$ is the temperature ratio parameter, $\Pr=\nu/\alpha$ is the Prandtl number and $Rd=16\sigma^*T_\infty^3/3kk^*$ is the radiation parameter. Note that for $c=0$ the above model corresponds to the two-dimensional flow whereas axisymmetric flow case is achieved by setting $c=1$.

The quantity of practical interest here is the local Nusselt number Nu_x defined by

$$Nu_x=\frac{xq_w}{k(T_w-T_\infty)}, \tag{11}$$

where $q_w=-k(\partial T/\partial z)_{z=0}+q_r$ is the wall heat flux. Now using dimensionless quantities from Eq. (6) into Eq. (11) we obtain

$$Re_x^{-1/2}Nu_x=-[1+Rd\theta_w^3]\theta'(0), \tag{12}$$

where $Re_x=U_wx/\nu$ is the local Reynolds number.

3. Numerical results and discussion

The solutions of Eqs. (7)–(9) with the boundary conditions (10) are obtained numerically by employing shooting approach with fifth-order Runge–Kutta method. The unknown initial conditions $f''(0), g''(0)$ and $\theta'(0)$ are estimated iteratively through Newton method. All computations are successfully performed in MATLAB with error tolerance of 10^{-6}. Our main focus in this section is to explore the behaviors of embedded flow parameters on the velocity and temperature distributions. For this purpose we plot Figs. 2–8 and prepare Tables 1 and 2 for the computational results.

For the validity of present simulations, we compared the numerical results of $f''(0), g''(0)$ and $\theta'(0)$ with Liu and Andersson [3] in a limiting sense. The results appear to be almost identical in all the cases as can be seen through Table 1. Influence of Deborah number K on the x- and y-components of velocity is sketched in the Fig. 2. The Deborah number is defined as the ratio of fluid relaxation time to its characteristic time scale. Relaxation time is the time taken by the fluid to gain equilibrium once the shear stress is imposed. The relaxation time is expected to be larger for the fluids having higher viscosity. Therefore an increase in K may be regarded as an increase in the fluid viscosity which restricts the fluid motion and hence the velocity decreases. Due to this reason the hydrodynamic boundary layer thins when K is increased. We also noted that change in the velocity fields f' and g' is larger in the three-dimensional flow when compared with the two-dimensional and axisymmetric flows.