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A fractional non-linear creep model for coal considering damage effect and experimental validation

Jianhong Kang*, Fubao Zhou*, Chun Liu, Yingke Liu

Key Laboratory of Gas and Fire Control for Coal Mines of Ministry of Education, School of Safety Engineering, China University of Mining and Technology, Xuzhou 221116, China

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ABSTRACT

Accurate prediction of coal's creep behavior is of great significance to coalbed methane extraction. In this study, taking into account the visco-elastic–plastic characteristics and the damage effect, a fractional non-linear model is proposed to describe the creep behavior of coal. The constitutive and creep equations of the proposed fractional non-linear model are derived via the Boltzmann superposition principle and discrete inverse Laplace transform. Furthermore, uniaxial creep tests under different axial stress conditions were carried out to validate the proposed model. It is found that the present model can describe the experimental data from creep tests with better accuracy than classical models. Particularly, the present model can predict the accelerating creep deformation of coal which classical models fail to reproduce. Finally, the parametric sensitivity analysis is performed to investigate the effects of model parameters on the creep strain. It is verified that the introduction of fractional parameters and damage factor in the present model is essential to accurate prediction of the full creep stage of coal.

1. Introduction

Coal and gas outburst is recognized worldwide as one of the potentially fatal hazards to be managed during the mining in deep coal mines [1]. The methane pre-extraction by drilling drainage boreholes in advance of mining is increasingly taken as an effective technique to prevent coal and gas outburst [2]. Meanwhile, the commercial extraction of coalbed methane has a significant long-term potential for the development and is currently well established in a number of countries [3]. The boreholes for methane extraction can be vertical boreholes drilled from the surface, or in-seam horizontal boreholes drilled from the underground entries [4]. However, in drilling industry coal and rock creep is one troublesome problem [5], which can cause the shrinkage, instability and collapse of boreholes as shown in Fig. 1, seriously affecting the extraction performance. Therefore, basic understanding and reliable modeling method of coal and rock creep is of great significance to methane extraction in practical engineering applications.

Study of creep behavior in the field of rock mechanics and mining science has been done extensively in the literature, involving theoretical, experimental and numerical methods. In general, the rheological behavior of rocks is typically characteristic of the viscosity, elasticity and plasticity. Many creep models have been proposed by researchers to describe the visco-elasto-plastic rheological behavior. The classical Burgers and Nishihara models are widely adopted in practical engineering due to its simplicity [6,7], but it cannot reproduce the accelerating creep stage. According to the damage mechanics of continuum, Wang [8] developed a new constitutive creep-damage model for rock salt. The internal variable-damage factor and its corresponding rheological deformation were used to demonstrate the creep-damage continuum macro-characteristics of rock salt. Yang et al. [9] investigated the time-dependent behavior of diabase using a non-linear creep model by connecting an instantaneous elastic Hooke body, a visco-elasto-plastic Schiffman body, and a non-linear visco-plastic body in series mode. Besides, through analysis of laboratory data Sone and Zoback [10] used the linear viscoelastic theory to evaluate the time-dependent deformation of shale gas reservoir rocks and its long-term effect on the in situ state of stress. Overall, despite increasing attention given to this subject as mentioned above, there still exists no simple and universal model for description of visco-elasto-plastic creep behavior in deep coal and rock seams.

On the other hand, during the last two decades fractional calculus due to its flexibility has gained much success in the description of rheological property of viscoelastic materials [11–16]. Since the pioneering work of Scott-Blair [17] who proposed a fractional element

* Corresponding authors. Tel.: +86 51683995053.
E-mail addresses: kang_jh@pku.edu.cn (J. Kang), zfbcumt@gmail.com (F. Zhou).

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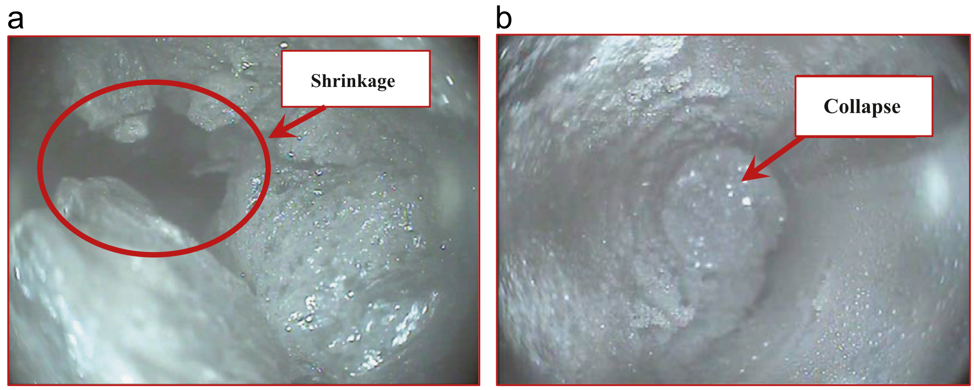

Fig. 1. Field photographs in two different methane drainage boreholes. (a) Borehole shrinkage at the depth of 13 m. (b) Borehole collapse at the depth of 25 m.

with analogy to the classical Hookean spring and Newtonian dashpot elements, various fractional-derivative models have been developed [18–20], the starting point of which is a combination of classical mechanical elements and Scott-Blair fractional elements. The main reason for this development is that fractional-derivative model can describe simply and elegantly the broad-band behavior of many viscoelastic materials with a small number of parameters [21]. Celauro et al. [22] used a fractional Burgers model to study the asphalt mixtures' behavior during their service life and showed that the fractional model fitted better with measured data. Yin et al. [23] adopted a fractional constitutive equation to simulate triaxial tests of geomaterials and found that the fractional model can reasonably describe the mechanical characteristics of geomaterials. More recently, Katicha et al. [24] presented a comprehensive linear viscoelastic characterization of asphalt concrete using fractional viscoelastic models. They concluded that fractional viscoelastic models are universal approximators and can be used to represent any spectrum. Besides, Goychuk [25] gave a general introduction to phenomenological description of linear viscoelasticity in complex media, in which the memory effect of viscoelastic materials was considered using the notion of fractional time derivative.

From an overview of the references, it can be found that (i) in spite of great importance to coalbed methane extraction, the study on creep behavior and constitutive model of coal is far less fruitful compared with that of rock; (ii) the fractional models can describe creep behavior of viscoelastic materials more flexibly and powerfully than classical models. Motivated by the above facts, in this paper we develop a fractional non-linear creep model of coal in which the damage effect at the stage of accelerating creep is taken into account. To validate the new model, uniaxial creep tests were carried out on coal specimens under different axial stress conditions. Besides, the effects of model parameters on the creep behavior of coal are discussed in detail.

2. Model development

2.1. Basic mechanical elements

In the linear viscoelastic theory, the mechanical analogy of a purely elastic solid can be represented by a spring as shown in Fig. 2(a). The relation between stress σ and strain ε for a elastic element obeys Hookean law, i.e.

$$\sigma(t) = E\varepsilon(t) \tag{1}$$

where E is the elastic modulus of spring. The mechanical analogy of a purely viscous fluid can be represented by a dashpot as shown in Fig. 2(b). The relation between stress σ and strain ε for a viscous element obeys Newtonian law, i.e.

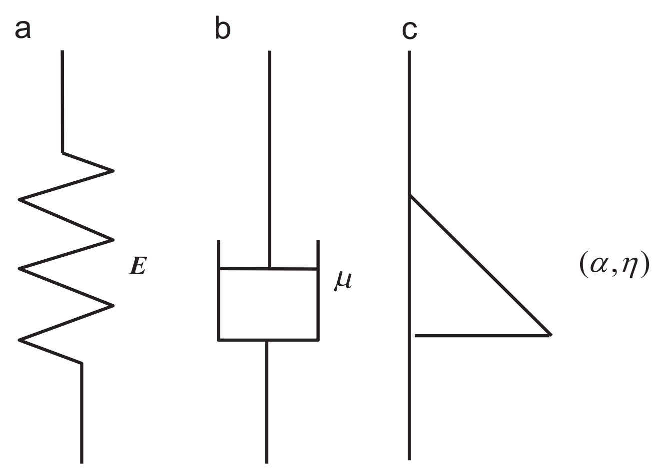

Fig. 2. The representations of basic mechanical elements. (a) The spring for a elastic solid. (b) The dashpot for a viscous fluid. (c) The fractional element for a viscoelastic body.

$$\sigma(t) = \mu \frac{d\varepsilon(t)}{dt} \tag{2}$$

where μ is the viscosity coefficient of dashpot.

From a phenomenological point of view, the mechanical response of a viscoelastic body should be intermediate between that of a purely elastic solid and that of a purely viscous fluid [26]. The stress of an elastic solid is proportional to the zeroth-order derivative of strain, while the stress of a Newtonian fluid is proportional to the first-order derivative of strain. Consequently, from a mathematical point of view, the simplest constitutive equation for a viscoelastic body may be described as

$$\sigma(t) = \eta \frac{d^\alpha \varepsilon(t)}{dt^\alpha}, \quad 0 < \alpha < 1 \tag{3}$$

which is known as the Scott-Blair fractional element [17] as shown in Fig. 2(c).

The differential operator d^α/dt^α in (3) denotes the α-order fractional derivative which is defined as [27]

$$\frac{d^\alpha \varepsilon}{dt^\alpha} = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{\varepsilon(\tau)}{(t-\tau)^\alpha} d\tau, \quad 0 < \beta < 1 \tag{4}$$

where $\Gamma(\cdot)$ is the Gamma function.

We note that two parameters α and η are required to characterize a fractional element, and for dimensional consistency the dimension of material parameter η should be [stress][time]$^\alpha$. It is also noteworthy that the definition of fractional derivative given in