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Chaotic vibrations of flexible curvilinear beams in temperature and electric fields

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ABSTRACT

In this paper regular and chaotic vibrations of flexible curvilinear beams with (and without) the action of temperature and electric fields are studied. Results obtained are based on the reduction of PDEs governing non-linear dynamics of straight and curvilinear beams to large sets of non-linear ODEs putting emphasis on reliability and validation of the results. In spite of the applied classical approaches to study bifurcational and chaotic dynamics, we have employed 2D and 3D Morlet wavelets and we have computed first four Lyapunov exponents. Numerous results are reported regarding scenarios of the transition from regular to chaotic vibrations including the occurrence of hyper-hyper chaos and deep chaos. Snap-through phenomena have been detected and analyzed, and the influence of boundary conditions of three types of the considered fields (mechanical, thermal and electrical) as well as of temperature on non-linear dynamics of the beam have been reported.

1. Introduction

There are numerous papers/monographs devoted to the study of dynamics of beams embedded either into an electric field or thermal field separately, and only a few papers address a simultaneous action of both fields on non-linear dynamics of beams.

The influence of electric field on free transverse vibrations of smart beams was studied by Krommer and Irschik [1].

Static and dynamic instabilities of the MEMs cantilever beam system subjected to weak and strong disturbances were investigated by Liu et al. [2]. They illustrated and analyzed period doubling, chaos and strange attractors for both open- and closed-loop cantilever systems subjected to strong disturbances.

Closed-form solutions of Euler–Bernoulli beams with singularities (flexural stiffness and slope discontinuities) are proposed in reference [3]. The continuity conditions are set into the flexural stiffness model and are included in the proposed procedure.

Zamianian and Khaden [4] studied microbeam dynamics under an electric actuation assuming that the microbeam midplane is stretched when it is deflected. Altering DC electric actuation in a microswitch system exhibits a saddle-node bifurcation point. Depending on the system parameters, periodic, quasi-periodic and pull-in instability can be achieved. The beam chaotic behavior is studied using Melnikov's approach.

Towfighian et al. [5] investigated the closed-loop dynamics of a chaotic electrostatic microbeam actuator with two wells of potential and with two distinct chaotic attractors. Period doubling, reverse period doubling, one-well and two-well chaos, as well as superharmonic resonances are reported, among others.

Barari et al. [6] applied variational iteration and parameterized perturbation methods to investigate the non-linear vibration of Euler–Bernoulli beams subjected to axial loads.

Shen et al. [7] analyzed the Euler–Bernoulli beam model where the absorbed heat flux on the beam surface depended on the beam deformation. The coupled thermal-structural analysis allowed them to predict a cantilever beam movement from eclipse with large incident angles of solar radiation.

Li et al. [8] studied non-linear equilibrium equations of the *slender* pinned-fixed Euler–Bernoulli beams regarding their buckling behavior.

A thin-walled composite beam including the interaction between structural deformations and incident heating was studied by Ko and Kim [9]. The beam model includes transverse shear deformation and rotary inertia, as well as primary and secondary

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warping effects. Steady-state thermal response was examined using the uncoupled analysis, whereas thermal flutter was studied by the coupled thermal-structural method.

Li et al. [10] analyzed the fundamental frequency of slender Euler beams embedded into the thermal field under various boundary conditions. The post-buckling behavior of functionally graded material beams with edge crack effects was investigated by Ke et al. [11]. Li et al. [12] analyzed thermal stability of nonlinear vibrations of a hybrid functionally graded Timoshenko beam with both clamped edges.

Gilat and Aboudi [13] applied the Lyapunov exponents methodology to study dynamic buckling of composite plates undergoing a sudden thermal or mechanical loading. The approach involving Lyapunov exponents to the quantitative analysis of continuous mechanical systems was applied in both works.

Wu [14] used Hamilton's principle to derive the equations of motion of a pinned beam with transverse magnetic fields and thermal loads. It was shown that the transient vibratory behavior of the beam was influenced by the magnetic and thermal loads; for example, the period of vibration increased with the increase of the magnetic fields and temperatures.

Pull-in instability of the double-clamped microscale beams actuated by a suddenly applied electrostatic force and subjected to non-linear squeeze film damping was investigated by Krylov [15] through monitoring the largest Lyapunov exponent evolution.

Li et al. [16] studied vibrations of functionally graded material beams with surface-bounded piezoelectric layers in thermal environment based on the Euler–Bernoulli beam theory. The beams were covered with piezoelectric layers and subjected to thermo-mechanical loadings. However, the authors reduced the problem to two sets of coupled ordinary differential equations. They showed that the temporal force produced in the piezoelectric layers by the voltage could efficiently increase the critical buckling temperature and the natural frequency.

Non-linear vibrations of the functionally graded material beam bounded with/without piezoelectric layers in a thermal environment were studied by Wang et al. [17] who analyzed pre/post-buckling phases of non-linear vibrations.

Yu et al. [18] studied free vibration of thermal post-buckled functionally graded material beams subjected to both temperature rise and voltage. They showed that three lower frequencies of the pre-buckled (buckled) beam decreased (increased) with the temperature rise, among others.

As it has already been mentioned, there are also works dealing with the problems of non-linear beam dynamics with thermal and electrical excitations. Non-linear vibrations of thermo-electrically post-buckled rectangular functionally graded piezoelectric beams were studied by Komijani et al. [19]. Thermo-electro-mechanical beam properties were graded across the beam thickness, and both in-plane and out-of-plane boundary conditions were considered. The effects of boundary conditions, beam geometry, actuator voltage, and thermal environment action were studied.

The general numerical approach used in this work, as employed to structural members, has already been presented in our earlier papers [21–27]. This type of the studied problem has been analyzed briefly in reference [25]. However, the mentioned short report concerned a straight beam.

Our work is organized in the following way. The flexible curvilinear beam model is introduced in Section 2, where also the types of heat boundary conditions are defined. Then the influence of boundary conditions is briefly illustrated in Section 3. Section 4 deals with the scenarios of transition into chaotic regimes. Chaotic vibrations of the curvilinear beam embedded into a temperature field are studied in Sections 5 and 6 for different types of heat boundary conditions using FFT (Fast Fourier Transform) and LE (Lyapunov Exponent) characteristic of different beam curvature and temperature magnitude. Section 7 illustrates the effect of beam curvature on the solution of the heat transfer equation, whereas the influence of the electric field on the beam dynamics is illustrated and discussed in Section 8. Concluding remarks are presented in Section 10.

2. Beam model

We consider a one-layer thin flexible curvilinear beam of length l, height h and geometric curvature $k_x = 1/R_x$, where R_x is the curvature radius. The beam is loaded through continuous load along the beam surface $q(x,t)$, acting in the normal direction to the middle beam surface (Fig. 1).

A mathematical model of the beam is based on the hypotheses of shallow shells introduced by Reissner and Vlasov. Namely, for shallow spherical shells the ratio of deflection f to the smallest shell plane dimension of $(f/a) \le 1/8$ (Reissner) or $(f/a) \le 1/5$ (Vlasov) and the geometry in space coincides with that of plane.

The mathematical model of the curvilinear beam is governed by a system of non-linear partial differential equations (PDEs) describing the motion of a beam element taking into account energy dissipation which is represented by the occurrence of damping coefficient ε. The non-dimensional form of PDEs regarding displacements is as follows

$$
\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} - k_x \frac{\partial w}{\partial x} + L_3(w,w) - \frac{\partial^2 u}{\partial t^2} = 0,\\
&\frac{1}{\lambda^2}\left\{ -\frac{1}{12}\frac{\partial^4 w}{\partial x^4} + k_x\left[\frac{\partial u}{\partial x} - k_x w - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - w\frac{\partial^2 w}{\partial x^2}\right]\right.\\
&\left. + L_1(u,w) + L_2(w,w)\right\} + q - \frac{\partial^2 w}{\partial t^2} - \varepsilon\frac{\partial w}{\partial t} = 0,\\
&L_1(u,w) = \frac{\partial^2 u}{\partial x^2}\frac{\partial w}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2}; \quad L_2(w,w) = \frac{3}{2}\left(\frac{\partial w}{\partial x}\right)^2 \frac{\partial^2 w}{\partial x^2};\\
&L_3(w,w) = \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2},
\end{aligned} \tag{1}
$$

where: $L_1(u,w)$, $L_2(w,w)$, $L_3(w,w)$ are the non-linear operators; $w(x,t)$ – normal deflection of the beam element; $u(x,t)$ – longitudinal displacement of the beam element; ε– coefficient of dissipation of the surrounding medium; E – Young modulus; h – height of the beam transversal cross section; γ– specific material gravity; g– Earth acceleration; k_x – geometric beam curvature; t – time; $q = q_0 \sin(\omega_p t)$ – external load; q_0– amplitude; ω_p – frequency.

Non-dimensional parameters are introduced in the following way:

$$
\bar{\lambda} = \frac{a}{h}; \quad \bar{w} = \frac{w}{h}; \quad \bar{u} = \frac{ua}{h^2}; \quad \bar{x} = \frac{x}{a}; \quad \bar{t} = \frac{t}{\tau}; \quad \tau = \frac{a}{p}; \quad p = \sqrt{\frac{Eg}{\gamma}};
$$

$$
\bar{\varepsilon} = \frac{a}{p}\varepsilon; \quad \bar{q} = \frac{qa^4}{h^4 E}; \quad \bar{k}_x = \frac{k_x a}{\lambda}.
$$

Bars over the non-dimensional quantities are omitted in Eq. (1). The following boundary conditions are applied: one beam end is

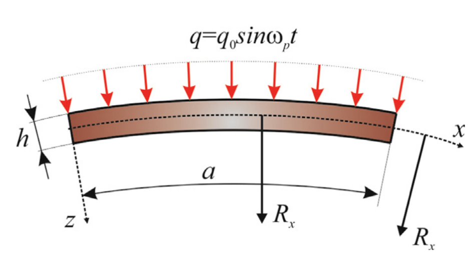

Fig. 1. Flexible curvilinear beam.