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Nonlinear vibration of fluid-conveying single-walled carbon nanotubes under harmonic excitation

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ABSTRACT

In this paper, the nonlinear vibration of a single-walled carbon nanotube conveying fluid is investigated utilizing a multidimensional Lindstedt–Poincaré method. Considering the geometric large deformation of the single-walled carbon nanotube and external harmonic excitation force, based on nonlocal elastic theory and Euler–Bernoulli beam theory, the nonlinear vibration equation of a fluid-conveying single-walled carbon nanotube is established. Analyzing the equation through the multidimensional Lindstedt–Poincaré method, and from the solvability condition of the nonlinear vibration equation, the cubic algebraic equation which indicates the amplitude–frequency relation is obtained. Based on the root discriminant of the cubic equation, the first order primary response of the pinned–pinned carbon nanotube is discussed. The relations among internal resonance, the amplitude and frequency of the external excitation force are analyzed in detail. When the external excite force frequency is around the first mode natural frequency, the first mode primary resonance occurs. If simultaneously the first two modes natural frequency ratio is around 3, internal resonance occurs and the internal resonance region depends on the amplitude of external excitation force.

1. Introduction

Carbon nanotubes (CNTs) have attracted considerable attention after their discovery [1]. Because of their unique hollow cylindrical geometry structure and remarkable mechanical and electrical properties [2], CNTs have substantial applications in gas storage [3], fluid storage, fluid transport [4], and drug delivery system [5–7].

As a small scale fluid–structure interaction system, the fluid-conveying CNTs show higher sensitivity to the vibration characteristics, and the research of flow-induced vibration and instability is of fundamental significance. Up till now, there are mainly three categories for investigating the mechanical characteristics of CNTs: experiment research, molecular dynamics simulations (MDS), and analysis based on continuum mechanics. As the radii of CNTs are generally vary from point several to tens of nanometers, the lengths are mostly in dozens of nanometers to microns, a controlled experiment to study mechanical properties of CNTs is very difficult. An MDS method requires an enormous amount of computational effort especially for large sized atomic systems. Thus this method can be applied only to study small sized systems. Elastic beam and shell models are the major continuum mechanic theories used in analyzing mechanical and vibration characteristics of CNTs. When the aspect ratio of CNTs is much larger than 10, the shear deformation and rotary inertia of the beam can be neglected and Euler–Bernoulli beam theory can be utilized [8,9]. For short CNTs or higher mode analysis, shear deformation and rotary inertia are important for the vibration of CNTs, and thus Timoshenko beam theory is needed [10–12]. Yoon et al. [8,9] researched the influence of internal moving fluid on free vibration and stability of CNTs using the classical Euler–Bernoulli model for both the supported and cantilevered systems. They found that the internal moving fluid could substantially affect resonant frequencies especially for longer CNTs of larger radius at higher flow velocity, and the critical flow velocity for structural instability or flutter could fall within the range of practical significance. The surrounding elastic medium can significantly reduce the effect of internal moving fluid on resonant frequencies. Khosravian and Rafii-Tabar [10] investigated the effect of non-viscous fluid flow on the vibration of multi-walled carbon nanotubes (MWCNTs) using both Timoshenko beam model and Euler beam model. The main conclusion has been that, compared to Euler classical beam model, the Timoshenko beam model predicts the loss of stability at lower fluid flow velocities. Based on Timoshenko beam theory, Hsu et al. [11] investigated the resonance frequency of chiral SWCNTs and found that the effect of shear deformation and rotary inertia is significant as the aspect ratio is low. Lee and Chang [12] researched the flexural

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vibration of the fluid-conveying SWCNT by the Timoshenko beam model and found that the influence of shear deformation and rotary inertia can be neglected when the aspect ratio is 60. Based on an elastic shell model, Yan et al. [13] studied the flow-induced instability of double-walled carbon nanotubes (DWCNTs). They pointed out that the critical flow velocity and loss of instability are significantly related to the van der Walls (vdW) interaction and the aspect ratio of DWCNTs.

As the size of CNTs is sufficiently small, the material's microstructure becomes more important and cannot be ignored anymore. Classical continuum elasticity, which is a scale independent theory, cannot predict the size effect and maybe no longer accurate enough for simplification of vibration characteristics of CNTs. The nonlocal continuum mechanics regard that the stress state at a given point is a function of the strain states of all points in the body which clarifies the scale effect in elasticity. The nonlocal continuum mechanics contains the information of the long-range forces between atoms, and the internal length scale is introduced into the constitutive equations simply as a material parameter [14]. Application of nonlocal continuum theory to nanotechnology was initially addressed by Peddieson et al. [15] to analyze the static deformations of beam structures based on a simplified nonlocal model [16]. Lee and Chang [17] studied the free transverse vibration of the fluid-conveying single-walled carbon nanotubes (SWCNTs) using nonlocal elastic theory. They found that the frequency and mode shape are influenced by the small length scale and the effect was more obvious as the flow velocity decreased, especially for the higher-order modes. Tounsi et al. [18] developed a more accurate equation of the vibration of SWCNTs conveying fluid. Wang [19] studied the dynamical behavior of DWCNTs conveying fluid accounting for the role of small length scale. It was demonstrated that the effect of small length scale on the critical flow velocities can be neglected. Zhen and Fang [20] found that the nonlocal effect is more obvious as the fluid velocity increase which is different from the above results, and interpreted the reason for the difference. Ghavanloo et al. [21] found that curved CNTs embedded in vicso-elastic medium are unconditionally stable even for a system with sufficiently high flow velocity. Moreover, some investigation on wave propagation, buckling and vibration of CNTs have been done based on nonlocal Timoshenko and higher order beam theories [22–27]. In the past decades, several other theories (e.g., gradient elasticity theory and modified couple stress theory) have been used to study the dynamical behaviors of fluid-conveying micro- and nanotubes. Wang [28] studied the reliability of various theoretical beam models via gradient elasticity theories for wave propagation analysis of fluid-conveying SWCNTs with either Euler–Bernoulli beam theory or Timoshenko beam theory and either stress or strain gradients. It is found that the combined strain/inertia gradient Timoshenko beam theory is more suitable for analyzing the dynamical behaviors of fluid-conveying nanotubes. Ke and Wang [29] investigated the vibration and instability of fluid-conveying DWCNTs based on the modified couple stress theory and Timoshenko beam theory.

In recent years, some investigations have been done on the nonlinear problems of CNTs [30–35]. Nonlinear vibrations of nanotubes have been studied in the case of a SWCNT [36] and in the case of DWCNTs [37] where geometric nonlinearity and simply supported boundary conditions are considered. The effect of the geometric nonlinearity and the nonlinearity of vdW forces on the transverse vibration of the DWCNTs conveying fluid and the interaction between two types of nonlinearities are investigated by Kuang et al. [38]. Ke et al. [39] studied the nonlinear free vibration of embedded DWCNTs based on Eringen's nonlocal elasticity theory and von Kármán geometric nonlinearity using the Timoshenko beam model. Rasekh et al. [40] investigated the influence of internal moving fluid and compressive axial load on the nonlinear vibration and stability of embedded CNTs. Considering geometric nonlinearity and nonlinear vdW force, Fang et al. [41] analyzed nonlinear vibration of DWCNTs. Adali [42] provided a variational formulation for MWCNTs and derived the natural boundary conditions at a free end using a nonlinear continuum model. Wang and Li [43] studied the nonlinear free vibration of nanotube with small scale effects embedded in viscous matrix.

Furthermore, due to large elastic deformation, nonlinear vibration of microelectromechanical systems (MEMS) and nanoelectromechanical systems (NEMS) appear in the practical engineering and play an important role in the design and analysis of MEMS/NEMS. Nayfeh and Balachandran have done much work on the nonlinear forced vibration and internal resonance of macro-systems [44–46]. But as to our knowledge, there are few investigations on the nonlinear vibration of fluid-conveying SWCNTs with external harmonic excitation, especially for the resonant characteristics with small scale effects. Considering these factors, investigation on nonlinear vibration properties of nanotube can provide a useful help for the design and analysis of MEMS/NEMS devices working at large amplitudes.

In this article, based on nonlocal elasticity theory and Euler–Bernoulli beam theory, the forced vibration and internal resonance of a SWCNT conveying fluid under harmonic excitation is researched utilizing the multidimensional Lindstedt–Poincaré (MDLP) method.

2. Equations

A schematic diagram of a fluid-conveying SWCNT embedded in elastic medium with two ends simply supported is shown in Fig. 1. In the analytical model, we suppose the internal fluid is an incompressible steady flow and the gravity effect is neglected. It is assumed that there is no tangential external loading along the axial and the circumferential directions of the SWCNT. In this case, the axial displacement of the SWCNT is negligible. From Ref. [20] we get the vibration equation of fluid-conveying SWCNT embedded in elastic medium based on nonlocal Euler–Bernoulli beam theory as follow:

$$\begin{aligned} &EI\frac{\partial^4 w(x,t)}{\partial x^4}+m\frac{\partial^2 w(x,t)}{\partial t^2}+m_f\left(u^2\frac{\partial^2 w(x,t)}{\partial x^2}+2u\frac{\partial^2 w(x,t)}{\partial x\partial t}+\frac{\partial^2 w(x,t)}{\partial t^2}\right)\\ &\quad+kw(x,t)-\frac{\partial}{\partial x}\left(N_x\frac{\partial w(x,t)}{\partial x}\right)(e_0a)^2\left[m\frac{\partial^4 w(x,t)}{\partial x^2\partial t^2}+\frac{\partial^2}{\partial x^2}(kw(x,t)\right.\\ &\quad\left.-\frac{\partial}{\partial x}\left(N_x\frac{\partial w(x,t)}{\partial x}\right)\right).+m_f\left(u^2\frac{\partial^4 w(x,t)}{\partial x^4}+2u\frac{\partial^4 w(x,t)}{\partial x^3\partial t}+\frac{\partial^4 w(x,t)}{\partial x^2\partial t^2}\right)\\ &\quad\left.+k\frac{\partial^2 w(x,t)}{\partial x^2}\right]=0, \end{aligned} \tag{1}$$

where $w(x,t)$ is the transverse displacements of the SWCNT along the x-axis at time t, E is the Young's modulus of the SWCNT, I is the moment of inertia, m and m_f are the mass of SWCNT and the mass of the fluid per unit length respectively, u is the flow velocity of the internal fluid. e_0a represents the effect of nonlocal elasticity and k is the Winkler constant of the surrounding elastic medium, N_x is the axial pressure.

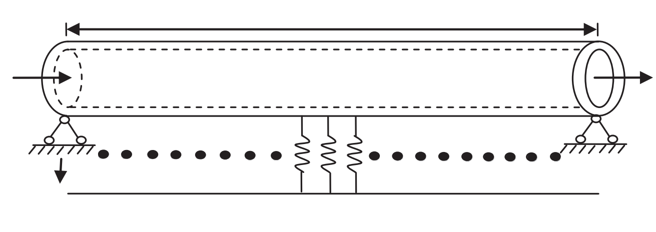

Fig. 1. A schematic diagram of a fluid-conveying SWCNT embedded in elastic medium.