International Journal of Non-Linear Mechanics 76 (2015) 56–63

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Mathematical model of inchworm locomotion

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ARTICLE INFO

Article history:
Received 4 April 2015
Received in revised form
22 May 2015
Accepted 23 May 2015
Available online 3 June 2015

Keywords:
Inchworm
Locomotion
Arching
Elastica

ABSTRACT

Inchworms are caterpillars. Their locomotion, involving arching of much of the central portion of their body length, has not been studied as extensively as the peristaltic locomotion of worms or the crawling locomotion of many other caterpillars. A mathematical model is developed to describe the shapes and bending strains of typical inchworm motions. The inchworm is assumed to travel in a straight line on a rigid horizontal substrate. Two basic types of cycles are considered. In Case I, the inchworm body arches and then reverses that motion in becoming flat again. In Case II, the body arches, then cantilevers upward, and then falls down to a flat shape. A continuum model based on an elastica is adopted. The results may be useful in the development of soft robots exhibiting an inchworm mode of motion.

1. Introduction

Inchworm locomotion involves alternate releasing and grasping of the front and rear sets of legs, and arching (sometimes called looping or inching) incorporating much of the animal's length (https://www.youtube.com/watch?v=cyasgr9mn3s). The mechanics of the locomotion of an inchworm has received relatively little attention compared to that of similar animals [1,2]. Some inchworm configurations from [3] are reproduced in Fig. 1.

The arching shape of an inchworm resembles the post-buckled shape of a continuous flexible beam with fixed (clamped) ends. A beam with such large displacements is often analyzed as an elastica, in which the beam is elastic and the bending moment is assumed to be proportional to the curvature [4–7]. In the present paper, this concept is utilized to develop a mathematical model of the shapes and bending strains of an inchworm.

Inchworms are not worms, but caterpillars [8]. Worms typically move using peristalsis, with axial (longitudinal) waves propagating along their bodies and utilization of frictional resistance with the substrate [9,10]. Caterpillars contain a number of sets of short legs: true (thoracic) segmented legs in the head section, and prolegs (unsegmented appendages) in the central and tail sections [11–14]. Caterpillar bodies are segmented, with intersegmental membranes connecting the segments [2]. Many caterpillars exhibit crawling motions that involve arching of a small part of the body, with the arched portion moving in a wave toward the head [8,11,15–23]. Inchworms do not contain some of the central prolegs that exist in crawling caterpillars, and inchworm locomotion includes arching that involves much of the length of the body (Fig. 1b).

Inchworms are the larvae of moths of the family Geometridae and are sometimes called loopers, spanworms, or measuring worms [11,21]. A cycle of motion begins as seen in Fig. 1(a), with the inchworm on the substrate. During the first phase (Phase 1) of the cycle, most of the central portion exhibits arching, as sketched in Fig. 1(b), with the head section remaining stationary (anchored) and the tail section moving toward the head. Two basic types of motion can occur during the second phase (Phase 2). For both, the tail section remains stationary. In Case I, the sequence of Phase 1 is reversed until the arching is eliminated. In Case II, the head moves away from the substrate till the central and head sections are straight or almost straight, as shown in Fig. 1(c), and then the raised (cantilevered) portion moves back toward the substrate until the end of the cycle when the configuration in Fig. 1(a) is attained again. These two parts of Phase 2 will be designated Phases 2A and 2B, respectively. Inchworm locomotion tends to be several times faster than crawling motion [11,21].

One potential application of the results of the present paper is in the burgeoning field of "soft robotics" [24–33]. In 2013, the International Workshop on Soft Robotics and Morphological Computation was held in Switzerland. In 2014, workshops on Soft Robots and on Soft Medical Robots were conducted during the IEEE International Conference on Robotics and Automation (ICRA) in Hong Kong, a workshop on Soft Robotics took place at the Robotics Science and Systems Conference (RSS) in Berkeley, soft robots were on display at the International Conference on Intelligent Robots and Systems (IROS 2014) in Chicago, and a new journal entitled Soft Robotics was created.

Flexible inchworm-type robots were described in [14,34]. Some papers use terms like "inchworm-like robots", "inchworm robot",

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http://dx.doi.org/10.1016/j.ijnonlinmec.2015.05.007
0020-7462/

or "inchworm motion" in their titles, but involve robots that crawl and do not exhibit any or much arching [35–40]. Some robots exhibit arching behavior but are not soft, often containing rigid segments connected by joints [18,41–47]. Further papers describing robots and other devices inspired by inchworm locomotion include [48–52].

Some robots resembling earthworms or caterpillars are used for purposes such as endoscopies, colonoscopies, and inspection of pipes [37,39,53–57]. A discussion of the effect of friction on earthworm-type robots is presented in [58].

Shape memory alloy (SMA) wires are sometimes employed to actuate the locomotion of robots imitating inchworms [2,14,19,27,34,59]. It is possible to predict the bending strains required for such actuations with the use of the type of analysis presented here.

The mathematical model will be formulated in Section 2. Results for Cases I and II will be described in Sections 3 and 4, respectively. Bending strains will be discussed in Section 5, followed by concluding remarks in Section 6.

2. Formulation

The arched shape of an inchworm on top of a rigid horizontal surface resembles that of the upward buckling of a thin, flexible, horizontal beam with clamped ends and subjected to end shortening (i.e., the ends are moved toward each other). Here the inchworm model is based on a uniform inextensible elastica, for which the material is linearly elastic and the bending moment is proportional to the curvature.

The lateral view of the undeformed model is shown in Fig. 2. The weight of the inchworm is neglected [21], so the orientation of the substrate and the direction of arching relative to the substrate are irrelevant. Dynamic effects (inertia, energy dissipation) are also neglected [27]. Configurations in a vertical plane are considered, with the inchworm assumed to move along a straight line.

The inchworm's depth (height when flat) is denoted D, which is the diameter if the cross section is circular. The inchworm's length is the sum of the lengths L_h of the head section, L of the central section, and L_t of the tail section. The head and tail sections are assumed to remain straight and to have a continuous slope at their connections with the central section, so that the analysis will only involve the central section. An arched shape of the centerline of

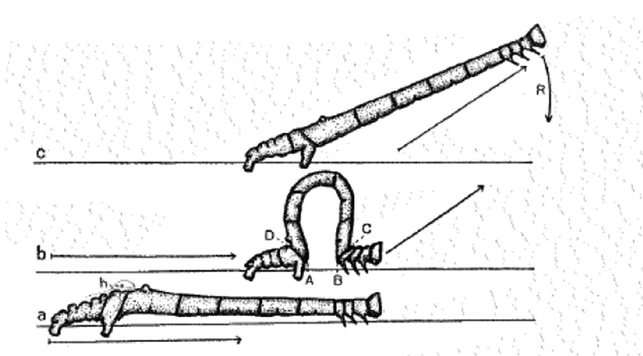

Fig. 1. Schematic of inchworm configurations, from [3] with permission: (a) initial flat configuration; (b) arched configuration; and (c) cantilevered configuration.

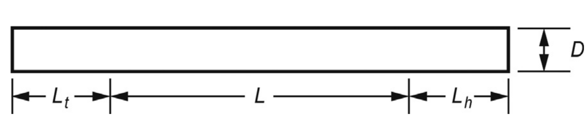

Fig. 2. Lateral view of model of inchworm in initial configuration.

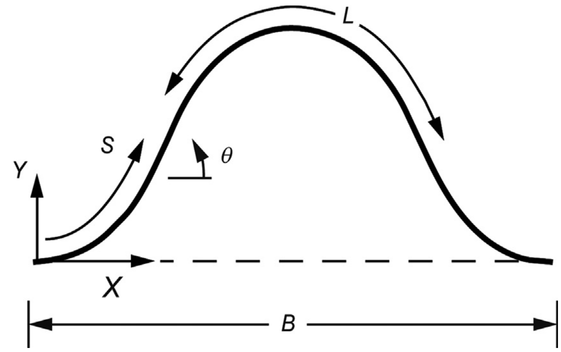

Fig. 3. Arched shape of centerline of central section of model.

this section is depicted in Fig. 3, with zero slopes at its ends. At arc length S, the inchworm has horizontal position $X(S)$, vertical position $Y(S)$, and rotation $\theta(S)$. The horizontal base length is B, with $B=L$ when the inchworm is lying on the substrate.

The analysis is conducted in terms of the following nondimensional quantities:

$$s=\frac{S}{L},\quad x=\frac{X}{L},\quad y=\frac{Y}{L},\quad b=\frac{B}{L},\quad d=\frac{D}{L},$$
$$\ell_t=\frac{L_t}{L},\quad \ell_h=\frac{L_h}{L},\quad m=\frac{ML}{EI},\quad p_o=\frac{P_oL^2}{EI}. \tag{1}$$

In Eq. (1), M is the bending moment (positive if counter-clockwise on a positive face), EI is the bending stiffness, and P_0 is the internal force in the horizontal direction (positive if compressive). Due to symmetry and vertical equilibrium, no vertical reaction forces will exist at the ends of the central section, and hence there will be no vertical component of internal force.

From geometry

$$\frac{dx}{ds}=\cos\theta,\quad \frac{dy}{ds}=\sin\theta. \tag{2}$$

For an inextensible elastica, the constitutive law and equilibrium provide the equations [5]

$$\frac{d\theta}{ds}=m,\quad \frac{dm}{ds}=-p_o\sin\theta. \tag{3}$$

The boundary conditions at $s=0$ are $x=y=\theta=0$, and at $s=1$ they are $x=b$ and $y=\theta=0$.

If a uniform inextensible elastica on a rigid horizontal foundation has fixed ends and the ends are displaced toward each other, the beam buckles upward into a symmetrical shape. These buckled shapes were computed here using the subroutines NDSolve and FindRoot in Mathematica [5]. For the nondimensional base length $b=0.32$, the resulting rotation $\theta(s)$ is plotted as the solid curve in Fig. 4. Rotations were computed for a number of base lengths between $b=0.3$ and $b=1$. The shapes $\theta(s)$ are similar to a sinusoid, and an approximation for the amplitude $f(b)$ was obtained using the subroutine Non linearModelFit.

The resulting approximation is

$$\theta(s)=f(b)\sin(2\pi s),\quad 0\le s\le 1 \tag{4}$$

with

$$f(b)=2.22-0.281b-4.533b^2+6.385b^3-3.494b^4\quad \text{if}\quad 0.3\le b\le 0.912 \tag{5}$$

and

$$f(b)=2.02\sqrt{1-b}\quad \text{if}\quad 0.912<b\le 1. \tag{6}$$