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Non-linear free vibration analysis of point supported laminated composite skew plates

A. Naghsh, M. Azhari *

Department of Civil Engineering, Isfahan University of Technology, 84156-83111 Isfahan, Iran

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ABSTRACT

The element-free Galerkin (EFG) method is employed to analyze the large amplitude free vibration of point supported laminated composite skew plates. The geometrical non-linearity is considered based on the von Karman's assumptions and the point support boundary conditions are satisfied through the use of Lagrange multiplier method and orthogonal transformation technique. Assuming a periodic solution and applying the weighted residual method, the non-linear governing equations are used, and the problem is solved by direct iteration technique. The obtained results are first validated against the available solutions of simply supported and clamped plates, and then the point supported plates are dealt with. To this end, four types of point support arrangements are considered and non-linear frequency ratios for different numbers of point supports, amplitude ratios, skew angles, aspect ratios and fiber orientations are calculated which may serve as benchmarks for future research.

1. Introduction

Laminated composite skew plates are widely used as the main components in aerospace, marine, and other modern industries. These materials allow high-strength structures with a minimum weight, thus forming thin plate components that are prone to large amplitude oscillations. Analysis of large amplitude free vibration of these plates has proven practically important in structural designs, and has recently received increasing attention of the researchers.

During vibration of a plate, by increasing the deflection, tensile membrane stresses develop in the thickness of the plate, which makes the plate stiffer and results in changing the frequencies and mode shapes. Consequently, in large amplitude vibrations, we have to deal with a non-linear problem, where the frequencies are amplitude-dependent.

Most studies reported on the non-linear vibration of laminated composite plates have been concerned with rectangular shaped plates, and have employed different methods for deriving and solving the equations of motion. However, a few works can also be found in the current literature on the large amplitude vibration of laminated composite skew plates, some of which are briefly touched on below. As a case in point, Singha and Ganapathi [1] dealt with non-linear free vibration of thin composite skew plates using finite element method. They investigated the effects of boundary conditions and various geometrical parameters of the plate on its frequency and mode shape. Singha and Daripa [2] reported large amplitude free vibration analysis of composite skew plates based on using 3 different approaches, namely (1) satisfying the equation of motion at the point of maximum displacement, (2) taking the weighted residual of the non-linear governing equation over a period, and (3) performing time history analysis. They showed that the frequencies obtained by second approach are in good agreement with those obtained from time history analysis and other available analytical solutions. Malekzadeh [3] presented non-linear frequencies of laminated composite skew thin plates having simply supported, clamped and mixed boundary conditions by using a differential quadrature method. Later Malekzadeh extended the differential quadrature method to free vibration analysis of moderately thick laminated skew plates based on the first order shear deformation theory [4].

Based on the available literature, all the works addressing the large amplitude free vibration of the composite rectangular or skew plates encompass the simply supported and clamped boundary conditions, plates with edges elastically restrained, or plates resting on elastic foundation. However, despite having practical significance, composite plates having edge or interior point supports have not been deservedly investigated through the large amplitude free vibration studies. It must be noted again that point supports are used in many cases to connect structural components, or are employed as an additional fulcrum in the plates having edge support to increase their loading capacity.

The linear vibration of point supported composite plates has been studied by several researchers. Narita and Iwato [5] presented free

* Corresponding author. Tel.: +98 3133913804; fax: +98 3133912700.
E-mail address: mojtaba@cc.iut.ac.ir (M. Azhari).

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vibration analysis of composite circular and elliptical plates resting on elastic or rigid point supports by employing the Ritz method. Liew [6] employed the Rayleigh–Ritz method and the Lagrange multiplier technique for free vibration analysis of angle-ply thin trapezoidal plates over point supports. Abrate [7] analyzed the free vibration of rectangular laminated composite plates with internal line supports and discrete point supports using the Rayleigh–Ritz method and the Lagrange multiplier technique. On the basis of three-dimensional elasticity considerations, Ye and Soldatos [8] studied free vibration of simply supported laminated plates and cylindrical panels with lateral surfaces point supports. They showed that for very thin plates on point supports, the two-dimensional classical theory provides very good results in accordance with three-dimensional analysis. Cheung and Zhou [9] investigated the free vibration of rectangular laminated composite plates with interior and edge point supports by using the Rayleigh-Ritz approach and employing a set of admissible functions. Setoodeh and Karami [10] reported a layer-wise finite element method to study the static, free vibration and buckling analysis of rectangular thick composite plates with elastic line and point supports. Kong [11] carried out free vibration analysis of rectangular thin isotropic and composite plates with various boundary conditions, including elastic point supported conditions by using the Rayleigh–Ritz approach. Bahmyari and Khedmati [12] studied free vibration of non-homogeneous moderately thick rectangular plates with point supports resting on elastic foundation by using Element-Free Galerkin Method. They employed Penalty method for imposing essential boundary conditions.

In recent years, mesh-free methods have been developed and successfully applied to the problems of solid mechanics, and the problems dealing with large deflection of plates. In mesh-free methods, the field approximation is based on a set of nodes scattered on the problem domain without any need for mesh generation or connectivity information among nodes. These methods have made it possible to overcome some drawbacks of finite element method (FEM), for example mesh distortion in large deformation problems and difficulties in remeshing of problems with moving discontinuities [13,14]. Mesh-free methods also suffer from their own drawbacks, i.e., construction shape functions is more time-consuming in contrast to FEM, and there are difficulties in handling essential boundary conditions. However, they often yield more accurate results and provide higher rate of convergence [15].

One of the popular approaches in the class of mesh-free numerical methods is the element-free Galerkin (EFG) method developed by Belytschko et al. [15]. In the EFG method, the field approximation is based on the moving least squares (MLS) approach, and essential boundary conditions are introduced into the analysis by means of Lagrange multipliers technique. This method has been used to solve various problems of shell and plate structures in the literature [16–18]. One of the advantages of EFG method in solving thin plate problems is that the derivatives of the displacement field do not appear as the unknown parameters in the analysis and thus the size of the total problem is reduced. This issue will become more important in problems that require iterative procedures.

In this paper, large amplitude vibration characteristics of thin laminated composite skew plates on point supports are investigated. The formulation is based on the classical plate theory in conjunction with the von Karman strain–displacement assumptions. The non-linear equations of motion are solved using the EFG method through employing direct iteration technique. Some numerical examples of rectangular and skew plates are presented for different point support conditions and the effects of number of point supports, skew angle, fiber orientation and aspect ratio on the non-linear frequency of laminated skew plates are reported.

2. Formulation

The plate considered here is a thin symmetrically laminated skew plate with length a and width b, resting on a set of arbitrarily located point supports (Fig. 1). The plate is composed of several orthotropic layers with the same thickness and mechanical properties. The fiber orientation in each layer is indicated by angle θ with respect to the x axis.

Based on the thin classical plate theory and considering the von Karman large deformation assumptions, the strains of the plate may be defined in the following form

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}_{0L}+\boldsymbol{\varepsilon}_{0NL}+z\boldsymbol{\kappa}=\begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y}+v_{0,x} \end{Bmatrix}+\frac{1}{2}\begin{Bmatrix} w_{,x}{}^2 \\ w_{,y}{}^2 \\ 2w_{,x}w_{,y} \end{Bmatrix}+z\begin{Bmatrix} -w_{,xx} \\ -w_{,yy} \\ -2w_{,xy} \end{Bmatrix}. \tag{1}$$

In the above relation, u_0 , v_0 are in-plane displacements of the plate mid-surface in x and y directions and w is out of plane displacement. $(\ )_{,x}$ and $(\ )_{,y}$ represent the partial differentiation with respect to x and y, respectively. $\boldsymbol{\varepsilon}_{0L}$ and $\boldsymbol{\varepsilon}_{0NL}$ are linear and non-linear part of the in-plane strains, respectively, and $\boldsymbol{\kappa}$ is the curvature change. For a symmetric laminated composite plate, the constitutive relations are

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix}=\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{bmatrix}\begin{Bmatrix} \boldsymbol{\varepsilon}_{0L}+\boldsymbol{\varepsilon}_{0NL} \\ \boldsymbol{\kappa} \end{Bmatrix}. \tag{2}$$

where $\mathbf{N}=\left\langle N_x \quad N_y \quad N_{xy}\right\rangle^{\mathrm{T}}$ is the vector of in-plane forces, and $\mathbf{M}=\left\langle M_x \quad M_y \quad M_{xy}\right\rangle^{\mathrm{T}}$ is the vector of bending and twisting moments defined per unit length. $\mathbf{A}$ and $\mathbf{D}$ are the axial and flexural rigidity matrices defined according to the properties and arrangement of layers through the thickness, and may be found in [19].

The equations of motion for large deformation free vibration of the plate may be derived by using principle of virtual work. The virtual work done by internal forces and inertia forces are denoted by W_s and W_k, respectively, and are expressed as

$$W_s=\int_A(\delta\boldsymbol{\varepsilon}_{0L}+\delta\boldsymbol{\varepsilon}_{0NL})^{\mathrm{T}}\mathbf{A}(\boldsymbol{\varepsilon}_{0L}+\boldsymbol{\varepsilon}_{0NL})\mathrm{d}A+\int_A(\delta\boldsymbol{\kappa})^{\mathrm{T}}\mathbf{D}(\boldsymbol{\kappa})\mathrm{d}A \tag{3}$$

$$W_k=\int_A\rho h(\delta u_0\ddot{u}_0+\delta v_0\ddot{v}_0+\delta w\ddot{w})\mathrm{d}A \tag{4}$$

where ρ is the mass per unit volume, h is the total thickness of the plate, A is the area and dots denote differentiation with respect to time t. After approximation of displacement field by EFG method and using strain equations (1), the virtual works may be expressed in a discretized form in terms of nodal displacement parameters. By setting the sum of the virtual works equal to zero and considering the virtual displacement to be arbitrary, the following system of equations of motion will be derived

$$(\mathbf{K}_L+\mathbf{K1}_{NL}+\mathbf{K2}_{NL})\boldsymbol{\Delta}+(\mathbf{M})\ddot{\boldsymbol{\Delta}}=\mathbf{0} \tag{5}$$

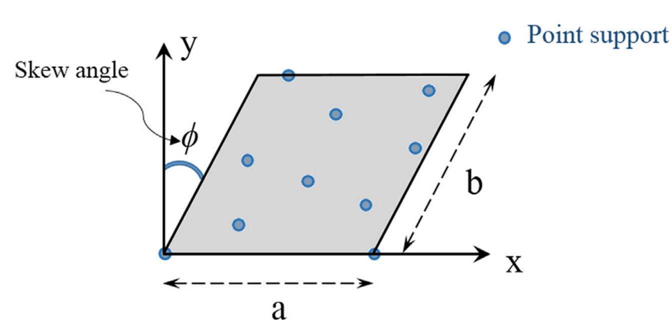

Fig. 1. The skew plate supported at discrete points.