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Surface and non-local effects for non-linear analysis of Timoshenko beams

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ABSTRACT

In this paper, we present a non-local non-linear finite element formulation for the Timoshenko beam theory. The proposed formulation also takes into consideration the surface stress effects. Eringen's non-local differential model has been used to rewrite the non-local stress resultants in terms of non-local displacements. Geometric non-linearities are taken into account by using the Green–Lagrange strain tensor. A C^0 beam element with three degrees of freedom has been developed. Numerical solutions are obtained by performing a non-linear analysis for bending and free vibration cases. Simply supported and clamped boundary conditions have been considered in the numerical examples. A parametric study has been performed to understand the effect of non-local parameter and surface stresses on deflection and vibration characteristics of the beam. The solutions are compared with the analytical solutions available in the literature. It has been shown that non-local effect does not exist in the nano-cantilever beam (Euler–Bernoulli beam) subjected to concentrated load at the end. However, there is a significant effect of non-local parameter on deflections for other load cases such as uniformly distributed load and sinusoidally distributed load (Cheng et al. (2015) [10]). In this work it has been shown that for a cantilever beam with concentrated load at free end, there is definitely a dependency on non-local parameter when Timoshenko beam theory is used. Also the effect of local and non-local boundary conditions has been demonstrated in this example. The example has also been worked out for other loading cases such as uniformly distributed force and sinusoidally varying force. The effect of the local or non-local boundary conditions on the end deflection in all these cases has also been brought out.

1. Introduction

The classical theory of hyperelasticity is used to solve a large number of problems in engineering, wherein the stress at a given point uniquely depends on the current values and possibly also the previous history of deformation and temperature at that point only. Deformation in this case is characterized by the deformation gradient or by an appropriate strain tensor, that is, it is fully determined by the first gradient of the displacement field. In modeling micro/nano structures where the size effect becomes prominent, for example, study of elastic waves when dispersion effect is taken to account and the determination of stress at the crack tip when the singularity of the solution is of concern, the classical theory cannot model the material behavior accurately.

The inhomogeneities present in any material at the microscopic scale influence its properties at the macroscopic scale: materials such as suspensions, blood flows, liquid crystals, porous media, polymeric substances, solids with microcracks, dislocations, turbulent fluids with vortices, and composites point to the need for incorporating micro-motions in continuum mechanical formulations [13]. There has been considerable focus towards the development of generalized continuum theories [19] that account for the inherent microstructure in such natural and engineering materials (see [36,15]). The notion of generalized continua unifies several extended continuum theories that account for such a size dependence due to the underlying microstructure of the material. A systematic overview and detailed discussion of generalized continuum theories has been given by Bazant and Jirasek [8]. These theories can be categorized as gradient continuum theories (see works by Mindilin et al. [45–47], Toupin [68], Steinmann et al. [13,34,66,37], and Casterzene et al. [52], Fleck et al. [20,63], Askes et al. [3–5]), microcontinuum theories (see works by Eringen [18,16,19]), Steinmann et al. [35,28], and non-local continuum theories (see works by Eringen [17], Jirasek [33], Reddy [54], and others [7,12,51,10]). Recently, the higher order gradient theory for finite deformation has been elaborated (for instance see [21,38,39,52]), within classical continuum mechanics in the context of homogenization approaches. A comparison of various higher order gradient theories can be found in [20]. A more detailed formulation of

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gradient approach in spatial and material setting has been presented in [35].

Classical continuum mechanics takes exclusively the bulk into account, nevertheless, neglecting possible contributions from the surface of the deformable body. However, surface effects play a crucial role in the material behavior, the most prominent example being surface tension. A mathematical framework was first developed by Gurtin [23] to study the mechanical behavior of material surfaces. The effect of surface stress on wave propagation in solids has also been studied by Gurtin [24]. The tensorial nature of surface stress was established using the force and moment balance laws. Bodies whose boundaries are material surfaces are discussed and the relation between surface and body stress examined in a recent work by Steinmann [64] and by Hamilton [25]. The surface effects have been applied to modeling two [31] and three-dimensional continua in the frame work of finite element method (see [32,14]). Similar studies on static analysis of nanobeams using non-local finite element models have been done by Mahmoud [43].

The focus of this work is on non-local non-linear formulation together with surface effects for static and free vibration analysis of Timoshenko beams. The non-local formulations can be of integral-type formulations with weighted spatial averaging or by implicit gradient models which are categorized as strongly non-local, while weakly non-local theories include for instance explicit gradient models [8]. Herein we consider a strongly non-local problem. The Timoshenko beam can be considered as a specific onedimensional version of a Cosserat continuum. Recently various beam theories such as Euler–Bernoulli, Timoshenko, Reddy, and Levinson beam theories were reformulated using Eringen's non-local differential constitutive model by Reddy [54]. The analytical solutions for bending, buckling and free vibrations were also presented in [54]. Various shear deformation beam theories were also reformulated in recent works by Reddy [55] using non-local differential constitutive relations. Similar works have been done to study bending, buckling and free vibration of nanobeams by Aydogdu [7], Civalek [12].

Eringen's non-local elasticity theory has also been applied to study bending, buckling and vibration of nanobeams using Timoshenko beam theory (see [40,60,72,48]). Numerical solutions were obtained by a meshless method. Two different collocation techniques, global (RDF) and local (RDF-FD), were used with multiquadrics radial basis functions by Roque et al. [58]. Static deformation of micro- and nano-structures was studied using non-local Euler–Bernoulli and Timoshenko beam theory and explicit solutions have been derived for deformations for standard boundary conditions by Wang et al. (see [71,70]). Analytical solutions for beam bending problems for different boundary conditions were derived using non-local elasticity theory and Timoshenko beam theory by Wang et al. [69]. Iterative non-local elasticity for Kirchhoff plates has been presented in [62]. Thai et al. [67] developed a non-local shear deformation beam theory with a higher order displacement field that does not require shear correction factors. Some explicit solutions involving trigonometric expansions are also presented recently for non-local analysis of beams [74]. A finite element framework for non-local analysis of beams has also been made in a recent work by Sciarra et al. [61]. Size effects on elastic moduli of plate like nanomaterials have been studied in [65].

Non-local elastic rod models have been developed to investigate the small-scale effect on axial vibrations of the nanorods by Aydogdu [6] and Adhikari et al. [1]. Free vibration analysis of microtubules based on non-local theory and Euler–Bernoulli beam theory was done by Civalek et al. [12]. Free vibration analysis of functionally graded carbon nanotube with various thickness based on Timoshenko beam theory has been investigated to obtain numerical solutions using the Differential Quadrature Method (DQM) by Janghorban et al. [30] and others (see [11,27,2]). Studies to understand thermal vibration of single wall carbon nanotube embedded in an elastic medium using DQM have also been reported in [49]. The recent studies have been towards the application of non-local non-linear formulations for the vibration analysis of functionally graded beams [53]. Analytical study on the non-linear free vibration of functionally graded nanobeams incorporating surface effects has been presented in [26,59,42]. The effect of non-local parameter, surface elasticity modulus and residual surface stress on the vibrational frequencies of Timoshenko beam has been studied in [73,41]. The coupling between non-local effect and surface stress effect for the non-linear free vibration case of nanobeams has been studied in [29]. The effect of surface stresses on bending properties of metal nanowires is presented in [75]. There has been some works on transforming non-local approaches to gradient type formulations [9]. Semi-analytical approach for large amplitude free vibration and buckling of non-local functionally graded beams has been reported in [50].

In this paper, we present a non-local non-linear finite element formulation for the Timoshenko beam theory. The proposed formulation takes into consideration the surface stress effects. Eringen's non-local differential model has been used to write the non-local stress resultants. Geometric non-linearities are taken into account by using Green–Lagrange strain tensor. Numerical solutions are obtained by performing a non-linear analysis for bending and free vibration cases. Simply supported and clamped boundary conditions have been considered in the numerical examples. A parametric study has been performed to understand the effect of non-locality and surface stresses on deflection and vibration characteristics of the beam. The solutions are compared with the analytical solutions available in the literature. The following Section 2 gives a background on Eringen's non-local theory. Section 3 gives the mathematical formulation for the non-local Timoshenko beam theory. The finite element formulation for the Timoshenko beam theory is explained in Section 4. In Section 5 numerical examples are presented together with parametric studies to demonstrate the effect of non-local and surface stresses on the bending and vibration characteristics of the beam.

2. Non-local theories

In classical elasticity, stress at a point is a function of strain at that point. Whereas in non-local elasticity, stress at a point is a function of strains at all points in the continuum. In non-local theories, forces between the atoms and internal length scale are considered in the constitutive equation. Non-local theory was first introduced by Eringen [19]. According to Eringen, the stress field at a point x in an elastic continuum not only depends on the strain field at that point but also on the strains at all other points of the body. Eringen attributed this fact to the atomic theory of lattice dynamics and experimental observation on phonon dispersion. The non-local stress tensor σ at a point x in the continuum is expressed as

$$\boldsymbol{\sigma} = \int K(|\mathbf{x}' - \mathbf{x}|, \tau)\mathbf{t}(\mathbf{x}')\, d\mathbf{x}' \tag{1}$$

where $\mathbf{t}(\mathbf{x})$ is the classical macroscopic stress tensor at point $\mathbf{x}$ and the kernel function $K(|\mathbf{x}' - \mathbf{x}|, \tau)$ represents the non-local modulus, $|\mathbf{x}' - \mathbf{x}|$ is the distance and τ is the material constant that depends on internal and external characteristic lengths.

Stress and strain at a point are related to each other by Hooke's law as

$$\mathbf{t}(\mathbf{x}) = \mathbf{C}(\mathbf{x}) : \boldsymbol{\varepsilon}(\mathbf{x}) \tag{2}$$

where t is the macroscopic stress tensor, $\boldsymbol{\varepsilon}$ is the strain tensor, $\mathbf{C}$ is the fourth-order elasticity tensor and ':' denotes double dot product. Eqs. (1) and (2) together form the non-local constitutive equations of Hookean solid. Constitutive equations can also be