



Non-linear thermoelasticity with energy limiters



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ABSTRACT

In this article we couple the previously developed theory of elasticity with energy limiters for modeling material failure with a description of thermal processes. We present a generalized formulation of the thermoelasticity with energy limiters. We illustrate the new theory via the analytically tractable example of uniaxial tension. The theory predicts the existence of the thermoelastic inversion point observed in experiments. Besides, it predicts the experimentally observed stiffening of rubberlike materials under heating. We find above all that the tensile strength increases as a result of the stiffening while the critical rupture stretch is almost unaffected by heating. It is hoped that the developed coupled theory can be useful for modeling failure in rubberlike solids.

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1. Introduction

Continuum thermodynamics has a long history yet its formulation is still open to debate [11,21,22,13,29,12,15,7]. We will not participate in the debate, however. Instead, we will consider the simplest formulation that couples thermal and mechanical processes in the course of large deformations of rubberlike solids. The latter theory of non-linear thermoelasticity was pioneered by Chadwick [3] and further considered in Refs. [4,16,14,9,1,17], for example.

The existing theories of thermo-mechanics of rubberlike solids consider the intact material behavior, in which the deformation description does not incorporate mechanical failure. However, real materials do fail and their constitutive equations should include a description of failure. In the context of purely mechanical deformation without thermal coupling a very simple account of material failure in the constitutive laws was proposed in a series of recent publications: [23,24,25,27,28]. The basic idea was to introduce an energy limiter in the expression for strain energy. Such limiter enforces saturation – *the failure energy* – in the strain energy function, which indicates the maximum amount of energy that can be stored and dissipated by an infinitesimal material volume during rupture. The limiter induces stress bounds in the constitutive equations *automatically*.

The mentioned approach of elasticity with energy limiters does not include the thermo-mechanical coupling. The purpose of the present work is to fill this gap and propose a variant of the non-linear thermoelastic formulation with energy limiters. It is worth emphasizing that such formulation is by no means unique and the

present one can be seen as a starting point. This new formulation is given in Section 3 after a brief review of non-linear thermoelasticity in Section 2. Then the account of thermoelastic incompressibility is given in Section 4. The latter issue was considered more or less implicitly in the previous literature and it is reasonable, perhaps, to introduce it explicitly and separately. In Sections 5 and 6 respectively we further specialize the constitutive model and apply it to the problem of homogeneous uniaxial tension that is tractable analytically. Discussion in Section 6 summarizes the work.

We mention, finally, that the developed theory predicts the existence of the thermoelastic inversion point observed in experiments. Besides, it predicts the experimentally observed stiffening of rubberlike materials under heating. Remarkably, we find that the tensile strength increases as a result of the stiffening while the critical stretch of rupture is almost unaffected by heating. It is hoped that the developed coupled theory can be useful for modeling failure in rubberlike solids.

2. Basic finite thermoelasticity

Material point that occupies position \mathbf{x} in the reference configuration moves to position $\mathbf{y}(\mathbf{x})$ in the current configuration of a deformable body Ω . Deformation in the vicinity of the material point is described by the deformation gradient tensor

$$\mathbf{F} = \text{Grad } \mathbf{y}. \quad (2.1)$$

The linear and angular momentum balance take the following forms accordingly:

$$\text{Div } \mathbf{P} + \mathbf{b} = \rho \mathbf{a}, \quad (2.2)$$

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$$\mathbf{P}\mathbf{F}^T = \mathbf{F}\mathbf{P}^T, \quad (2.3)$$

where the divergence operator is calculated with respect to referential coordinates \mathbf{x} ; \mathbf{P} is the 1st Piola–Kirchhoff stress tensor; \mathbf{b} is the body force pure unit reference volume; and ρ and \mathbf{a} are the referential mass density and acceleration vector correspondingly.

Balance of linear momentum on the body surface $\partial\Omega$ reads

$$\mathbf{P}\mathbf{n} = \bar{\mathbf{t}}, \quad (2.4)$$

where $\bar{\mathbf{t}}$ is a prescribed traction per unit area of the reference surface with the unit outward normal \mathbf{n} .

Alternatively to (2.4) a surface boundary condition can be imposed on placements

$$\mathbf{y} = \bar{\mathbf{y}}, \quad (2.5)$$

where the barred quantity is prescribed on the surface $\partial\Omega$.

Initial conditions are

$$\mathbf{y}(t=0) = \mathbf{y}_0, \quad \mathbf{v}(t=0) = \mathbf{v}_0, \quad (2.6)$$

where \mathbf{v} is the velocity vector and \mathbf{y}_0 and \mathbf{v}_0 are prescribed in Ω .

Equation of the energy balance takes form

$$\dot{e} = \mathbf{P} : \dot{\mathbf{F}} - \text{Div } \mathbf{Q} + R, \quad (2.7)$$

where e is the internal energy per unit reference volume; \mathbf{Q} is the Piola–Kirchhoff heat flux; and R is a heat source.

Instead of the internal energy we use the Helmholtz free energy per unit reference volume

$$\psi(\mathbf{F}, \theta) = e - \theta\eta, \quad (2.8)$$

where η is the entropy per unit reference volume and θ is the absolute temperature.

Differentiating (2.8) with respect to time we obtain

$$\frac{\partial\psi}{\partial\mathbf{F}} : \dot{\mathbf{F}} + \frac{\partial\psi}{\partial\theta}\dot{\theta} = \dot{e} - \dot{\theta}\eta - \theta\dot{\eta}. \quad (2.9)$$

Substitution of the rate of the internal energy from (2.9) into (2.7) yields the energy balance in the form

$$\theta\dot{\eta} = (\mathbf{P} - \frac{\partial\psi}{\partial\mathbf{F}}) : \dot{\mathbf{F}} - (\eta + \frac{\partial\psi}{\partial\theta})\dot{\theta} - \text{Div } \mathbf{Q} + R. \quad (2.10)$$

Let us restrict further considerations by the following general constitutive laws:

$$\mathbf{P} = \frac{\partial\psi}{\partial\mathbf{F}}, \quad (2.11)$$

$$\eta = -\frac{\partial\psi}{\partial\theta}. \quad (2.12)$$

We note that in this case the entropy rate is

$$\dot{\eta} = -\frac{\partial^2\psi}{\partial\theta\partial\mathbf{F}} : \dot{\mathbf{F}} - \frac{\partial^2\psi}{\partial\theta^2}\dot{\theta}. \quad (2.13)$$

Substitution of (2.11)–(2.13) in (2.10) finally yields

$$c\dot{\theta} = -\text{Div } \mathbf{Q} + \theta \frac{\partial^2\psi}{\partial\theta\partial\mathbf{F}} : \dot{\mathbf{F}} + R, \quad (2.14)$$

where

$$c = -\theta \frac{\partial^2\psi}{\partial\theta^2} > 0, \quad (2.15)$$

is the specific heat capacity.

Balance of energy on the body surface $\partial\Omega$ reads

$$-\mathbf{Q}\mathbf{n} = \bar{\mathbf{q}}, \quad (2.16)$$

where $\bar{\mathbf{q}}$ is a prescribed heat flux through the unit area of the reference surface.

Alternatively to (2.16) a surface boundary condition can be imposed on temperature

$$\theta = \bar{\theta}. \quad (2.17)$$

Initial condition on the temperature might be necessary for the transition processes

$$\theta(t=0) = \theta_0, \quad (2.18)$$

where θ_0 is prescribed in Ω .

To complete the boundary value problem it remains to formulate the heat conduction equation in the Duhamel form, for example,

$$\mathbf{Q} = -J \mathbf{F}^{-1} \boldsymbol{\kappa} \mathbf{F}^{-T} \text{Grad } \theta, \quad (2.19)$$

where $J = \det\mathbf{F}$ and $\boldsymbol{\kappa}$ is the spatial thermal conductivity tensor.

We note that (2.19) is a generalization of the Fourier's constitutive law of heat conduction – see, for example, Holzapfel [9].

We note also that the second law of thermodynamics (in the Clausius–Planck form) requires the thermo-elastic processes to obey the following constraints:

$$D = \mathbf{P} : \dot{\mathbf{F}} - \dot{\psi} - \eta\dot{\theta} \geq 0, \quad (2.20)$$

$$\mathbf{Q} \cdot \text{Grad } \theta \leq 0. \quad (2.21)$$

The first constraint – the dissipation inequality – is obeyed by constitutive laws (2.11)–(2.12). The second constraint is obeyed by constitutive law (2.19) in which the spatial thermal conductivity tensor is positively definite.

3. Finite thermoelasticity with energy limiters

In this section, we incorporate a failure description in the non-linear thermoelasticity by using energy limiters. We start with one energy limiter. In this case the Helmholtz free energy per unit reference volume takes form

$$\psi(\mathbf{F}, \theta, \alpha) = \psi_f - H(\alpha) \psi_{te}(\mathbf{F}, \theta), \quad (3.1)$$

$$\psi_f = \psi_{te}(\mathbf{1}, \theta_0), \quad (3.2)$$

$$\psi_{te}(\mathbf{F}, \theta) \rightarrow 0, \quad \text{when } \|\mathbf{F}, \theta\| \rightarrow \infty, \quad (3.3)$$

where ψ_f and $\psi_{te}(\mathbf{F}, \theta)$ designate a constant bulk failure energy and a thermoelastic free energy respectively; $H(z)$ is a unit step function, i.e. $H(z) = 0$ if $z < 0$ and $H(z) = 1$ otherwise; $\mathbf{1}$ is a second-order identity tensor; and $\|\mathbf{F}, \theta\| = \mathbf{F} : \mathbf{F} + \theta^2$, for example.

The switch parameter $\alpha \in (-\infty, 0]$ is defined by the evolution equation

$$\dot{\alpha} = -H(\varepsilon - \psi_{te}/\psi_f), \quad \alpha(t=0) = 0 \quad (3.4)$$

where $0 < \varepsilon < 1$ is a dimensionless precision constant.

The physical interpretation of (3.1)–(3.4) is straight: material response is thermoelastic as long as the free energy is below its limit, ψ_f . When the limit is reached, then the free energy remains constant for the rest of the deformation process, thereby making material healing impossible. Parameter α is not an internal damage variable (like in damage mechanics) and it functions as a switch: if $\alpha = 0$ the process is thermoelastic, and if $\alpha < 0$, the material is irreversibly damaged and the thermoelastic energy is dissipated.

In presence of the switch parameter α , the free energy increment reads

$$\dot{\psi}(\mathbf{F}, \theta, \alpha) = \frac{\partial\psi}{\partial\mathbf{F}} : \dot{\mathbf{F}} + \frac{\partial\psi}{\partial\theta}\dot{\theta} + \frac{\partial\psi}{\partial\alpha}\dot{\alpha}. \quad (3.5)$$

The last term in (3.5) can be calculated based on (3.1)

$$\frac{\partial\psi}{\partial\alpha}\dot{\alpha} = -\delta(\alpha)\dot{\alpha}\psi_{te}, \quad (3.6)$$

where $\delta(\alpha)$ is the Dirac delta.

We notice that (3.6) in fact vanishes for all values of α . The case of $\alpha \neq 0$ follows immediately from the definition of Dirac's delta. In

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