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Non-linear forced periodic oscillations of laminates with curved fibres by the shooting method

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ABSTRACT

Large-amplitude forced vibration, before damage onset, of variable stiffness composite laminated plates with curvilinear fibres are studied. The fibre paths considered change linearly in relation to one Cartesian coordinate. The plates are rectangular and with clamped edges. The displacement field is modelled by a third order shear deformation theory and the equations of motion, in the time domain, are obtained using a *p*-version finite element method. The in-plane inertia is neglected, still taking into consideration the in-plane displacements, and the model is statically condensed. The condensed model is transformed to modal coordinates in order to have a reduced model with a smaller number of degrees-of-freedom. A shooting method using fifth-order Runge–Kutta method, as well as adaptive stepsize control, is used to find periodic solutions of the equations of motion. Frequency-response curves of composite laminates with different curvilinear fibre angles and various thicknesses are plotted and compared. Tsai–Wu criterion is employed in order to predict the damage onset. When it is detected that damaged started, the continuation method is interrupted and no further points of the response curve are computed. The reason behind this interruption is that the model does not include the effects of damage. Examples of bifurcations are presented and studied in detail, using projections of trajectories in a phase plane and Fourier spectra. The time histories and frequency spectra of steady-state stresses are plotted for VSCL plates with different fibre angles. The steady-state stresses are also displayed for bifurcated branches of the solutions.

1. Introduction

Variable stiffness composite laminated (VSCL) plates can be manufactured by changing the thickness of the laminate (adding or removing layers), connecting a stiffener to the laminate, changing the density of fibres, or, as in our case, by curving the fibres. Many of the papers allocated to analyses of VSCL plates are given in Ref. [1].

Here, a reference curvilinear fibre path is introduced as a function of horizontal coordinate x. Fibre paths in each ply reproduce this reference fibre path, with a shift in y axis. If instead of the curvilinear fibre path, a straight fibre path is chosen, the laminate will be a constant stiffness composite laminate (CSCL), a traditional fibrous composite.

Many works on different mechanical properties and optimisation of VSCL plates with different methods have been considered in Refs. [1,2]. Works especially on linear vibration of VSCL plates with curvilinear fibres are given in [3,4]. Static or dynamic deflections and stresses, under the effect of impacts and in transient responses, of VSCL plates with curvilinear fibres are investigated in Ref. [5–8]. Damage onset of such VSCL plates is studied in [6,9]. Behaviours of such VSCL plates in forced and free vibrations are studied respectively in Ref. [10] and Refs. [11,12]. In papers [3–12], diverse *p*-version finite elements with hierarchical basis functions are used; Refs. [7,8] used Newton-Raphson to obtain static solutions of VSCL plates subjected to constant forces. Ref. [10] includes studies about transient and periodic forced vibration of VSCL plates with curvilinear fibres using Newmark method. In Ref. [11], the harmonic balance method is applied to study the periodic free vibration of such panels. Other references have used Newmark method to solve the equations. We note that Newmark method based procedures alone, as the one employed in Ref. [10], do not allow to recognise bifurcations and instabilities in forced oscillations.

Unlike CSCL, VSCL plates have local strengths that vary along the plate; tailoring the fibre orientations to improve damage and failure properties of composites is one of the goals of using VSCL plates [13–16]. Ref. [15] studied postbuckling first-ply failure response and onset of delamination to estimate interlaminar stresses in VSCLs. Initiation of delamination is addressed in [17] by studying the response to impacts and the compression after impact. An example of design tailoring problem (the pressure pillowing of a fuselage VSCL panel) is given in [16] with the goal of maximizing the load carrying capacity. The analyses of Refs. [15–17] were carried out using the commercial finite element software

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Abaqus. An optimization approach is employed in [18] to optimize the strength around a circular hole, with Tsai–Wu failure criterion, in a VSCL plate. Refs. [19,20] investigated in-plane and buckling responses of VSCL plates and considered that, because the laminates were thin, the in-plane failure strains would be an order of magnitude larger than the buckling strains. Consequently, considerations for the in-plane strength failure were avoided. In studies on the vibration of laminates, failure analyses is more often than not neglected (an exception is Ref. [21], where it is arrived at the conclusion that in a moderately thick plate the material usually fails before the maximum deflection reaches the magnitude of the thickness). The authors believe that, especially when a moderately thick laminate is considered, it is interesting to investigate if failure appeared due to the vibrations.

Frequency response curves are fundamental to understand the non-linear vibratory characteristics of a structure [22]. The use of reduced order models and the investigation of bifurcations in dynamic analysis of panels have been well studied in Refs. [23–26]. However, to the best of the authors' knowledge, no study on the non-linear forced, periodic vibration of VSCL plates, including detection of bifurcations and instabilities, has been performed yet. This target is achieved in this paper. We intend to apply a p-version finite element model, based on a third-order shear deformation theory (TSDT), to VSCL plates with curvilinear fibres. The p-version finite element model, identified here as the "full model", can be condensed statically [27], leading to a model entitled as "statically condensed model". The number of degrees of freedom (DOF) of the full model or of the statically condensed model is reduced using modal coordinates [28]. Periodic solutions of the plate under harmonic loads, in the non-linear regime, are calculated using the shooting method [29–34] and frequency response curves are defined, in some cases with secondary branches that appear due to bifurcations. Some illustrative examples are studied in more detail using projections of trajectories in a phase plane (that, will be designated here as "phase plane plots"), time history plots and Fourier spectra. In bifurcation branches and in different VSCL plates, the variation of stresses in time and their frequency spectra are given. To facilitate the prediction of damage onset in VSCL plates under harmonic loads, Tsai–Wu failure criterion [35–37] is utilized. If a more accurate prediction is required, a criterion that takes into account the failure mode, like delamination, debonding, matrix failure or fibre failure [36,38], may be required. The aim of using Tsai–Wu criterion in our analyses is to prevent investigations of questionable interest, with a model that does not include the consequences of damage. We are going to show that thicker plates experience damage onset at lower non-dimensional vibration displacement amplitudes than thin plates. Furthermore, we will see that in the case of higher order modes, damage can start at somewhat low vibration amplitudes.

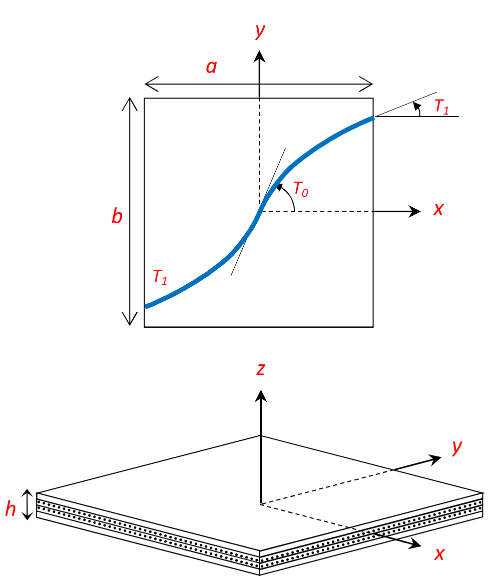

Fig. 1. A symmetric laminate including a reference curvilinear fibre path in a flat ply.

2. Non-linear forced vibration of VSCL plates

First, a rectangular plate is modelled by a p-version finite element in the time domain considering large deflections, using the so called von Kármán strain–displacement relations; the corresponding theoretical model is designated as the full model. Then a technique named *static condensation* [27] is applied to decrease the number of degrees of freedom (statically condensed model). The condensed model is converted to a reduced model using the *modal summation method* [28] (reduced model).

2.1. Full model of VSCL plates

A rectangular laminated plate ($a \times b \times h$), symmetric about its middle plane, in Cartesian coordinates with the origin located at the middle of the flat plate, and a reference curvilinear fibre path in a flat ply are displayed in Fig. 1. Other fibres, with the same curvature as the reference fibre, are placed by the shifting method, with shifts in y direction throughout each ply. With this technique, the fibre angle does not change in y direction, it only changes in x direction as $\theta_i(x) = 2(T_1 - T_0)|x|/a + T_0$, where i is the number of the lamina, T_0 the fibre angle at centre and T_1 the fibre angle at edges. Taking $T_1 = T_0$ results in a constant stiffness ply with straight fibres, but if $T_1 \neq T_0$ a variable stiffness ply with curvilinear fibres is obtained. A symmetric laminate with $2k$ layers may be shown as $[\langle T_0, T_1 \rangle^1, \ldots, \langle T_0, T_1 \rangle^i, \ldots, \langle T_0, T_1 \rangle^k]_{sym}$.

Based on a third-order shear deformation theory (TSDT) [39,40], the displacements of a generic point are related to the middle plane displacements and rotations of normals to the middle plane by

$$u(x,y,z,t) = u^0(x,y,t) + z\phi_x(x,y,t) - \frac{4z^3}{3h^2}\left(\phi_x(x,y,t) + \frac{\partial w^0(x,y,t)}{\partial x}\right),$$
$$v(x,y,z,t) = v^0(x,y,t) + z\phi_y(x,y,t) - \frac{4z^3}{3h^2}\left(\phi_y(x,y,t) + \frac{\partial w^0(x,y,t)}{\partial y}\right),$$
$$w(x,y,z,t) = w^0(x,y,t) \qquad (1)$$

with ϕ_x and ϕ_y defined as the rotations of the transverse normal about the y and x axes, respectively. Using the p-version finite element method (FEM) [41], the displacement components are related to the generalised displacements $\mathbf{q}$ by hierarchical basis functions [42]

$$\begin{Bmatrix} u^0(x,y,t) \\ v^0(x,y,t) \\ w^0(x,y,t) \\ \phi_x(x,y,t) \\ \phi_y(x,y,t) \end{Bmatrix} = \begin{bmatrix} \mathbf{N^u}(x,y)^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{N^u}(x,y)^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{N^w}(x,y)^T & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{N}^{\phi_x}(x,y)^T & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{N}^{\phi_y}(x,y)^T \end{bmatrix} \begin{Bmatrix} \mathbf{q_u}(t) \\ \mathbf{q_v}(t) \\ \mathbf{q_w}(t) \\ \mathbf{q}_{\phi_x}(t) \\ \mathbf{q}_{\phi_y}(t) \end{Bmatrix} \qquad (2)$$

where $\mathbf{N^i}(x,y)^T$, $\mathbf{i} = \mathbf{u}, \mathbf{w}, \boldsymbol{\phi_x}, \boldsymbol{\phi_y}$ are transpose of vectors composed of bi-dimensional in-plane, out-of-plane and rotational shape functions, the details of which can be found in [41–43]. Each vector $\mathbf{N^i}(x,y)$ has $p_i^2 = p_i \times p_i$ bi-dimensional elements, namely, p_u^2, p_w^2, p_ϕ^2. So, the generalised displacements vector has dimension $2 \times p_u^2 + p_w^2 + 2 \times p_\phi^2$ and this is the number of degrees of freedom (DOF) of the full model.