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Anti-plane shear waves in a fibre-reinforced composite with a non-linear imperfect interface

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ABSTRACT

The propagation of non-linear elastic anti-plane shear waves in a unidirectional fibre-reinforced composite material is studied. A model of structural non-linearity is considered, for which the non-linear behaviour of the composite solid is caused by imperfect bonding at the “fibre–matrix” interface. A macroscopic wave equation accounting for the effects of non-linearity and dispersion is derived using the higher-order asymptotic homogenisation method. Explicit analytical solutions for stationary non-linear strain waves are obtained. This type of non-linearity has a crucial influence on the wave propagation mode: for soft non-linearity, localised shock (kink) waves are developed, while for hard non-linearity localised bell-shaped waves appear. Numerical results are presented and the areas of practical applicability of linear and non-linear, long- and short-wave approaches are discussed.

1. Introduction

Elastic waves propagating in heterogeneous solids can undergo the effects of non-linearity and dispersion. Non-linearity may arise through geometrical, physical or structural mechanisms (e.g., Lur'e [48]). We study a problem for which the non-linear behaviour of a composite is associated with imperfect bonding conditions at the interface between constitutive components. This is an example of structural non-linearity, with the non-linearity directly related to the presence of a microstructure. Dispersion on the other hand can be classified as geometrical or structural. Geometrical dispersion is typical for wave-guides and finite-size bodies (e.g., waves in beams and plates). Structural dispersion may be caused by the heterogeneity of a composite solid, with successive reflections and refractions of local waves at the matrix–inclusion interfaces leading to scattering of the overall wave field.

Non-linearity induces a pumping of energy form the low- to the high-frequency part of the spectrum, with higher-order modes generated and continuous localisation of energy occurring, making the wave front steeper. In contrast, dispersion provides scattering of energy and decreases the slope of the wave front. When non-linearity and dispersion act together, they may balance the influence of each other [42]. In such a case, stationary non-linear waves of permanent shape and velocity can propagate. Non-linear strain waves play an important role in the mechanical behaviour of composite materials and structures. An increase in non-linearity leads to the formation of localised solitary waves. This process is accompanied by essential strain amplitude growth, possibly resulting in the development of local plastic zones and/or cracks. Therefore, non-linear dynamic effects can become a crucial factor affecting strength and durability of engineering structures.

In many cases, non-linear elastic moduli of heterogeneous solids are very sensitive to the properties of microstructure (see, for example, Zaitsev et al. [82]). Measuring the characteristics of non-linear waves enables detection of very small variations of the internal texture of the medium at a level not possible within a linear framework Zumpano and Meo, [83], Polimeno and Meo [65]. This provides the possibility of developing new, more precise, methods of the acoustic diagnostic and non-destructive testing in engineering, geophysics, biomechanics and other areas dealing with heterogeneous materials and structures.

The propagation of non-linear strain waves in elastic solids has been intensively studied. For a comprehensive review of the subject we refer to the books by Jeffrey and Engelbrecht [38], Maugin [51], Samsonov [73], Erofeev [29], and Porubov [66]. Many authors considered homogeneous systems, with dispersive properties mainly determined by geometrical factors. At the same time, the effect of structural dispersion, related to the scattering of non-linear waves by the microstructure, were not studied in great detail.

The influence of the microstructure can be modelled by allowing the elastic medium additional internal degrees of freedom, an idea originally proposed over 100 years ago by Cosserat and Cosserat [21]

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and Le Roux [44]. Following Cosserat theory, a number of higher-order continuum models were developed by Mindlin [53], Sun et al. [77], Herrmann and Achenbach [37]. More recently, similar approaches were adopted to describe the propagation of non-linear strain waves in miscrostructured solids [27,28,68,67,69]. A recent review of the modelling of heterogeneous media in terms of internal variables was presented by Berezovski et al. [12,13].

From a mathematical viewpoint, the aforementioned approaches supplement the constitutive equations of motion with some additional higher-order gradient terms accounting for the effects of dispersion. The coefficients at the dispersive terms represent what might be thought of as phenomenological parameters. In some cases these may be determined experimentally; however, for most industrial materials their magnitudes remain unknown. An alternative way to predict the influence of microstructures is provided by the asymptotic homogenisation method (AHM). According to this approach, physical fields in a spatially periodic heterogeneous medium are represented by a two-scale asymptotic expansion in powers of a small parameter $\eta = l/L$, where l is the size of the unit cell and L is the typical wavelength. This leads to a decomposition of the final solution into global and local components; the latter are evaluated from a recurrent sequence of cell boundary value problems (BVPs). Application of the volume-integral homogenising operator allows us to obtain a homogenised constitutive equation that describes the macroscopic behaviour of the medium. It is important to note that the coefficients of the homogenised equation (so-called *effective* moduli) are evaluated based on information about the properties of the components and the geometry of the microstructure. Thus, in contrast to Cosserat-type approaches, the homogenised model incorporates data about the internal composition of the material.

From its conception, the AHM was intended for the determination of quasi-static properties of heterogeneous media and structures (e.g., Bensoussan et al. [10], Sanchez-Palencia [74], Bakhvalov and Panasenko [9], Kalamkarov et al. [39]). Taking into account higher-order terms, with respect to η, extended the area of applicability of the homogenised models and provided a mechanism to predict the effect of structural dispersion [15,17,32,3,76,8]. Non-local effects resulting both from high-anisotropy and high-contrast of composite structures were studied by Cherednichenko et al. [18], Smyshlyaev [75], Soubestre and Boutin [76]. It should be noted that macroscopic dynamic equations obtained by the AHM are valid only in the long-wave case, when $l \ll L$. Recently, Craster et al. [22] have shown a subtle analogue between the long-wave asymptotic procedures underlying approximate formulations for periodic media and for functionally graded wave-guides. The conventional AHM seems to be a counterpart of the classical theories for thin plates, shells and rods. A theoretical framework for the asymptotic theories of long wave motion in plates and layered media was developed by Rogerson et al. [70,72] Rogerson and Prikazchikova [71], Lutianov and Rogerson [49], Mukhomodyarov and Rogerson [57]. Homogenisation of non-linear dynamic problems was considered by Andrianov et al. [7,5].

When the wavelength of a travelling signal decreases and becomes comparable to the size of the microstructure, a heterogeneous elastic solid exhibits a complicated sequence of pass and stop frequency bands. In the literature, they are also referred to as *phononic* bands (by analogy with the photonic bands arising for electromagnetic and optical waves in heterogeneous dielectric media). Thus, the composite plays the role of a discrete wave filter. If the frequency falls within a stop band, a stationary wave is excited and neighbouring heterogeneities (e.g. particles) vibrate in alternate directions. At a macrolevel, the amplitude of the global wave attenuates exponentially, so no propagation is possible. Phononic bands can be theoretically predicted using the Floquet-Bloch approach [14]. This approach has been documented in the book by Brillouin [16], 2nd ed. and utilised by many authors (see, for example, Movchan et al. [56,54,55], McPhedran et al. [52] and references therein).

Craster et al. [24,25,23] and Nolde et al. [61] proposed a generalisation of the AHM making it suitable for the analysis of high-frequency waves. The key point was to choose a zero-order approximation, not at the quasi-static limit ($\eta \to 0$), but at the edges of the high-frequency phononic bands.

In this present paper, we apply the AHM to the modelling of anti-plane shear waves propagating in a fibre-reinforced composite material with imperfect interface bonding between the matrix and fibres. For engineering materials, the properties of the interface may be subjected to various factors, such as the presence of thin coating layers, chemical reactions or mechanical damages. From the mathematical viewpoint, the effect of imperfect bonding can be predicted by assuming that the displacement jump across the interface is related to the interfacial stress by a certain cohesion function. This approach is general and can describe different types of interfaces independently of the physical reasons of the debonding.

In the simplest case, the cohesion function is assumed linear, the interface then acting like an elastic spring. The spring-type interface model was originally proposed by Goland and Reissner [33]. In the theory of composites, it was first introduced by Mal and Bose [50] and later employed by a number of authors. Variational formulations for the imperfect bonding conditions were presented by Hashin [36], Lipton and Vernescu [47]. Limiting cases of very soft and very stiff interfaces were analysed by Benveniste and Miloh [11]. Needleman [58,59], Tvergaard [79,80], Espinosa et al. [31,30] considered more sophisticated cohesion functions and simulated various scenarios of the debonding process. While reducing the cohesion, the stress field (supported by the interface) increases in magnitude, achieves a maximum, and ultimately falls to zero when complete separation occurs. It is therefore possible to track the evolution of the debonding process from its initial onset to complete separation and subsequent formation of voids. Non-linear interfaces were considered by Levy and Dong [46], Levy [45], Nguyen and Levy [60]. An asymptotic simulation of the imperfect bonding was presented by Andrianov et al. [2,4].

It should be noted that most of the interface models include a number of phenomenological parameters: the maximal interfacial traction, characteristic lengths of the interfacial displacements, interface shear-to-normal strength ratio, etc. Such quantities can not usually be identified a priori. At the same time, due to evident experimental difficulties. there is still very little effort to measure cohesive laws in real materials. As a successful example, we refer to Tan et al. [78] who developed an experimental approach to determine the microscopic cohesive law in composite high explosives.

In our present contribution we specifically study a weakly non-linear interface, with a cohesion function represented by a power series expansion in terms of non-dimensional displacement jumps. The paper is organised as follows. In Section 2, an asymptotic model of the imperfect bonding is proposed and the input BVP introduced. In Section 3, the higher-order asymptotic homogenisation procedure is developed and the macroscopic non-linear wave equation obtained. In Section 4, the analytical solution for stationary non-linear strain waves is derived in terms of elliptic functions. The analysis of the obtained results and numerical examples are presented in Section 5. Section 6 is devoted to the conclusions.

2. Asymptotic model of the imperfect bonding and input BVP problem

Let us consider a unidirectional fibre-reinforced composite consisting of an infinite matrix $\Omega^{(1)}$ and a periodic square array