International Journal of Non-Linear Mechanics 83 (2016) 59–64

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Flow of a micropolar fluid due to a porous stretching sheet and heat transfer

M. Turkyilmazoglu

Department of Mathematics, Hacettepe University, 06532-Beytepe, Ankara, Turkey

ARTICLE INFO

Article history:
Received 15 March 2015
Received in revised form
11 April 2016
Accepted 11 April 2016
Available online 12 April 2016

Keywords:
Micropolar fluid
Stretching surface
Mass transfer
Heat transfer
Exact solution

ABSTRACT

The present work investigates the micropolar fluid flow due to a permeable stretching sheet and the resulting heat transfer. Unlike the existing numerical works on the flow phenomenon in the literature, the prime interest here is to analytically work out shape of the solutions and identify whether they are unique. Indeed, unique solutions are detected and presented in the exact formulas for the associated boundary layer equations. Temperature field influenced by the microrotation is also mathematically resolved in the cases of constant wall temperature, constant heat flux and Newtonian heating. To discover the salient physical features of many mechanisms acting on the considered problem, it is adequate to have the analytical velocity and temperature fields and also closed-form skin friction/couple stress/heat transfer coefficients, all as given in the current paper. For instance, the practically significant rate of heat transfer is represented by a single formula valid for all three temperature cases.

1. Introduction

Owing to significant engineering and practical applications of non-Newtonian fluids like molten polymers, blood, fluid suspensions, food and cosmetic, researchers have recently paid much attention to one such fluid, named micropolar fluid, after the model offered by Eringen [1]. The traditional velocity vector is coupled with a microrotation vector in the flow equation of micropolar fluids. Hence, micropolar fluids are capable of understanding the flow phenomena at microscale and rotation as inferred from the books by Lukazewics [2] and Eringen [3].

Pioneering work of Crane [4] has lead to many subsequent research activities on stretching bodies, since it finds practical applications, for instance, in textile industries in polymer processing and in manufacturing process of glass sheets. Literature is now full of such numerical studies, to cite a few are Wang [5], Siddheshwara and Krishna [6], Nazar et al. [7,8], Ishak et al. [9], Chen [10], Ishak [11] and Bhattacharyya et al. [12]. Micropolar fluid flow phenomenon with variable heat flux effects was analyzed in Ishak et al. [13] and the porous medium case was investigated in Rosali et al. [14]. Analytical means of working out the solution of flow over stretching surfaces were also exploited, see for instance, Kelson and Desseaux [15] and the recent articles by Turkyilmazoglu [16,17] and [18,19] amongst many others.

Although, in the case of Newtonian fluid there are many publications dealing with the exact solution, as far as the fluid flow problem over a stretching sheet regarding the micropolar fluid effects is concerned, it has not been treated analytically in an aim to find exact solutions, except perhaps the perturbation work in [15]. This is due to the coupled nature of governing micropolar flow motion containing highly nonlinear terms. Therefore, people are believed to be diverted to numerical means. However, it was very recently proved in [20] that it is quite possible to achieve closed-form solutions for the micropolar fluid flow motion over a shrinking sheet as for the traditional Newtonian flow. As a consequence, the appearance of dual solutions over a shrinking sheet was successfully explained through the exact flow domain. The prime motivation here is hence to investigate whether exact analytical solutions can also be found for the flow of micropolar fluid over a porous stretching sheet. Such solutions may also help us resolve the temperature field exactly, when the wall temperature or the heat flux is constant as well as when the Newtonian heating is applied. Indeed, as opposed to the existing numerical solutions in the literature, exact solutions were found for both flow and temperature fields. Moreover, such micropolar fluid solutions are proved to be unique, unlike the double solution nature of the corresponding viscous flow for the shrinking surface. Furthermore, closed-form analytical expressions for the skin friction coefficient, couple stress coefficient and Nusselt number are obtained, which are very helpful to examine diverse physical properties of micropolar fluids in engineering applications of non-linear mechanics, as such in biological fluids, in crystals and in lubricants. One may also refer to the recent pulsating flow work of [21] and other physical applications as studied in [22], to understand the role of porous stretching sheet in micropolar theory and

E-mail address: turkyilm@hacettepe.edu.tr

http://dx.doi.org/10.1016/j.ijnonlinmec.2016.04.004
0020-7462/

heat transfer in micropolar liquid. Finally, it is believed that the analytical results presented here will shed light upon the further research to be conducted on micropolar fluids and their not yet discovered physical features, such as the slip velocity and temperature jump mechanisms.

2. Formulation of the problem

Sufficient number of sources in the literature has successfully formulated the physical problem of micropolar fluid flow motion over a stretching surface. For the sake of being concise and exerting more of the efforts to the derivation of exact solutions, the final similarity equations of coupled nature that govern the boundary layer flow of micropolar fluid motion and also temperature field (relevant to constant wall temperature) (see for example, [8,11]) are given by

$$(1+K)f''' + ff'' - f'^2 + Kh' = 0,$$

$$(1+K/2)h'' + fh' - f'h - K(2h + f'') = 0,$$
$$\theta'' + Pr\, f\theta' = 0, \tag{2.1}$$

which are supplemented together with the boundary conditions

$$f(\eta) = s,\quad f'(\eta) = 1,\quad h(\eta) = -mf''(\eta),\quad \theta(\eta) = 1 \text{ at } \eta = 0,$$
$$f'(\eta) \to 0,\quad h(\eta) \to 0,\quad \theta \to 0 \text{ as } \eta \to \infty. \tag{2.2}$$

It should emphasized here that the above system is different from that of [20], since the latter dealt with the shrinking wall case. To briefly state, η is a scaled boundary layer coordinate, $f'(\eta)$ is the similarity velocity component, $h(\eta)$ is the similarity microrotation or angular velocity, s is the wall permeability parameter with $s < 0$ for the wall suction and $s > 0$ for the wall blowing, material parameter is K and m represents a measurement for the concentration of microelements, respectively. It is further well-documented that weak microelement concentrations without the ability of rotation of microelements are represented by $m=0$, weak microelement concentrations with stress tensor having no antisymmetric component are denoted by $m=1/2$, and the boundary layer flow in turbulent state means $m=1$. Moreover, θ is the scaled fluid temperature and Pr is the usual Prandtl number. It is noted that when K is set to zero the classical Newtonian viscous fluid flow and heat problem over a stretching sheet is recovered.

3. Exact analytical solutions

The system of Eqs. (2.1)–(2.2) was numerically solved before by many researchers to explore the physical properties of micropolar fluid flow motion. Instead, full solutions in analytic are targeted here.

3.1. Solution of the flow and microrotation fields

In the stretching sheet problem concerning the Newtonian fluid flow, pursuing the exact solution of Crane [4], Pop and Na [23] obtained nice exact formulae representing the boundary layer flow in different flow configurations. Since the micropolar fluid flow equations (2.1)–(2.2) convey the traditional Newtonian flow character in the limit $K \to 0$, the aforementioned realistic solutions should also cover effects of non-Newtonian behavior, at least for some particular cases. Bearing this in mind, it can be assumed that system (2.1)–(2.2) possesses solutions of exponential type

$$f(\eta) = s + \frac{1 - e^{-\lambda\eta}}{\lambda},$$
$$h(\eta) = -mf''(\eta) = m\lambda e^{-\eta\lambda}, \tag{3.1}$$

where, the parameter λ is to be determined so that the system (2.1) and boundary conditions (2.2) are all satisfied. Thus, to meet the infinity boundary conditions λ must be positive. Moreover, the following algebraic system of equations results from a direct substitution of (3.1) into (2.1)

$$-1 - s\lambda + (1 + K(1-m))\lambda^2 = 0,$$
$$2m(1 + s\lambda - \lambda^2) - K(2 + m(-4 + \lambda^2)) = 0. \tag{3.2}$$

Solutions of (3.2), in turn, lead to

$$\left(m = 1/2,\ \lambda = \frac{s + \sqrt{4 + 2K + s^2}}{2 + K}\right),$$
$$\left(m = \frac{8(1+K)^2}{\left(s + \sqrt{4 + 4K(3 + 2K) + s^2}\right)^2},\ \lambda = \sqrt{\frac{2}{m}}\right). \tag{3.3}$$

By inspection, it is observed that exponential type physical solutions (3.1) exist when $m=1/2$ and also for the rest of m being dependent upon the physical parameters K and s. Since K is assigned to be positive in practical computations, a unique solution is anticipated for $s \in R$. This is quite distinct from the viscous fluid flow induced by a shrinking surface for which dual solutions of exponential type (3.1) were found see, for instance, the recent articles by Bhattacharyya et al. [24] and Turkyilmazoglu [20] and also the references therein. It should also be pointed out that the existence of unique/multiple solutions of the “auxiliary” equations (3.2) is a necessary reason, but not sufficient in general. Even though, in the present analysis the proved is that the current problem admits a unique exponential solution, it does not automatically negate the existence of any other form of analytic solution. However, as aforementioned, the provided solutions are physical since they collapse onto the well-known physical solutions previously worked out by famous researchers in the field. Moreover, in the limit of infinite material parameter $K \to \infty$, values in (3.3) turn out to the explicit asymptotic expressions

$$(m = 1/2,\ \lambda \to 0,\ s \in R),$$

$$(m \to 1,\ \lambda \to \sqrt{2},\ s \in R). \tag{3.4}$$

It is worthy of remarking here that the existence domain of parameter λ for any fixed values of K and s enables one to obtain the scaled skin friction coefficient $f''(0)$ and the scaled couple stress coefficient $h'(0)$, which are both covered from the above exact formula in the form

$$f''(0) = -\lambda,\quad h'(0) = -m\lambda^2.$$

We should remind that the true local skin friction parameter based on the wall skin friction will be affected by the microrotation and hence is given by

$$(1 + (1-m)K)f''(0).$$

The local couple stress parameter is also accordingly adjusted.

3.2. Solution of the temperature field

The exact solution expressing the velocity and microrotation fields via (3.1) and (3.3) dominates the temperature field of constant wall temperature. Taking this into consideration and solving the energy equation, the scaled temperature field θ takes the