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A non-linear vibration spectroscopy model for structures with closed cracks

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ABSTRACT

Ensuring an uninterrupted service in critical complex installations requires parameter health monitoring of the vibrating structures. Tools for monitoring structural modifications through changes in the measured dynamic responses are necessary in order to detect the advent and evolution of cracks before the occurrence of catastrophic failures. It is shown, both theoretically and experimentally, that the equation for the modes of vibration of a structure with closed (breathing) cracks and whose surfaces enter into contact during vibration can be modeled using the Hertz contact theory. The damping chosen is a fractional order derivative to investigate the order matching the experimental data. A perturbation solution technique, combining the Multiple Time Scales and Lindsted–Poincaré methods, has been employed to construct analytical approximations to the resulting non-linear equation of vibration. A 3D finite element model of the structure has been employed to compute the eigenvalues of the sound structure, providing a means to validate the measured resonance frequencies and also allowing the visualization of the modal deformations thus giving greater insight into the physics of the problem.

1. Introduction

The objective of this study is to develop a model that can capture the characteristics of a vibrating structure and also follow the changes in its measured dynamic response which can lead to the identification of structural modification like the advent of micro- and macrocracks and their growth. The evolution of the response can eventually facilitate the follow-up of the degradation of the structural flaw allowing for a correction of the defect before the occurrence of a catastrophic failure. This topic touches the non-destructive evaluation (NDE) and parameter health monitoring of vibrating structures for ensuring an uninterrupted service in critical complex installations like aircraft (rotor dynamics), ships, steel bridges, and sea platforms.

In micro-inhomogeneous materials (microdamaged, with micro-cracks), the non-linear hysteretic stress/strain relationship behavior has been modeled using mesoscopic mechanical elements with stress (strain) dependent parameters. To this end, the phenomenological Preisach–Mayergoysz (PM) spaces have been employed to model the hysterical non-linearity of the mesoscopic elements composed of individual mechanical elements (internal contacts) that open and close at different stress/strain level thresholds [1–3]. A micro-contact based theory similar to the PM model for micro-damaged materials has also been proposed in Refs. [4,5]. The material considered had a large number of isotropic oriented penny-shaped cracks with rough internal surfaces in contact. The Hertzian contact forces for surface roughness and adhesion were modeled using the Johnson–Kendal–Roberts adhesion theory. Similarly, the Preisach hysteretic formalism has been applied to the more complex but general case of the Hertz–Mindlin system [6] (model used for predicting dry effective elastic moduli in unconsolidated sediment) dealing with frictional and elastic interactions of two spheres in contact subjected to a varying oblique force [7].

When the samples are small in size and have well defined geometries, the non-linear resonance ultrasound spectroscopy (NRUS) characterization technique [8] can be employed. The NRUS technique with an interaction model based on non-linear constitutive equations, has been applied successfully to solve an inverse problem of defect imaging using experimental data [9].

Most engineering materials contain small crack-like defects (microcracks) which they can also spontaneously develop during service under fluctuating loads. Macrocracks result from the accumulation of damage due to fatigue loading. Failure can occur from repeated fluctuating stresses or strains, i.e., fatigue, causing catastrophic failure. Fatigue loading is also believed to cause accumulation of damage in osteoporotic bones in terms of microcracks [10] whose coalescence may lead to initiation of macrocracks that

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result in the catastrophic failure of bone [11]. This may consequently occasion fractures especially at common osteoporotic fracture sites (the wrist, femoral neck, the hip and the spine).

The presence of a breathing macrocrack in a structure has been known to introduce a local flexibility. This has been modeled as a continuous flexibility by using the displacement field in the vicinity of the crack [12].

In this study we consider a single mechanical, structural vibration mode approach in which the restoring force attached to the mass is made up of a spring in parallel with a deformation force due to the presence of the crack. The crack is modeled by supposing that the two crack surfaces enter into contact during vibration and that the contact obeys the basic Hertzian contact law [13–17] with the effect of adhesion due to the van der Waals forces (two solid surfaces brought into close proximity to each other experience attractive van der Waals forces [18]) neglected. In parallel to the spring is a dashpot whose damping behavior obeys a fractional-order derivative whose order is sought by matching the model to vibration spectroscopy experimental data.

The resulting mechanical system equation drawn from the developed model is a non-linear ordinary differential equation (NLODE) with a fractional-order derivative. NLODE is complicated to solve exactly. Numerical methods, such as finite difference methods and multi-grid methods, can be employed, but they only provide approximate numerical solutions that do not easily provide direct insight as into the physics of the problem related to the structural parameters. NLODE of dynamical systems is often solved by searching approximate periodic solutions using different perturbation techniques. These approximate analytical solutions are close to the true ones. Their advantage over the numerical solutions is that they are analytical expressions (instead of just a long list of numbers) that enable one to gain some insight into the underlying physics of the problem. Some solutions are based on the combination of linearization of the governing NLODE using the popular method of harmonic balance [19], with some approaches, unlike the classical harmonic balance method, performing linearization prior to proceeding with harmonic balancing [20].

Most of the perturbation methods, such as the Krylov–Bogoliubov–Mitropolskii (KBM) [21], Variational iteration methods [22,23], Averaging methods [24], Method of Matched Asymptotic Expansions [25,26], Renormalization method [27–29], Method of Multiple Scales [30,31], are developed in the time domain.

The NLODE with fractional-order derivative model equation is solved herein by combining the Multiple Time Scales and the Lindstedt Poincaré perturbation methods [32] in the frequency domain. The solutions are sought in this domain for the ease of observation and interpretation of the non-linear phenomenon.

A vibration spectroscopy experiment employing drinking glasses, shell-like solid structures, in which cracks can be produced easily by thermal shocks, has been set up to validate the model. The experimental observation employs off the shelf piezoelectric excitation transducer and sensor.

2. Theory

The elastic deformation of the impacting crack surfaces (Fig. 1) is accounted for using the Hertz contact theory for two connected Hookean bodies where the restoring contact force between them is given by

$$f(\delta)=\kappa\delta^{\chi}, \tag{1}$$

wherein δ is the local deformation (penetration) and κ is a constant dependent on the elastic and geometric properties of the contact surfaces. Both κ and the exponent χ can be determined

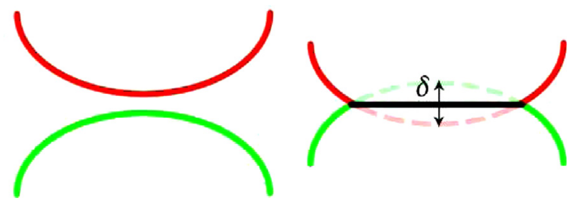

Fig. 1. The contact problem model geometry of a cracked structure showing the deformation kinematics before and after impact.

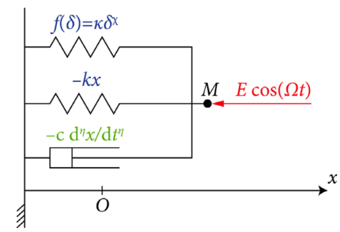

Fig. 2. The non-linear vibration model of a cracked structure considered as a single degree of freedom mechanical model with Hertzian contact and a damping force of the fractional-order.

experimentally. In the literature the exponent for planar contacts of various materials and surface textures/state [33,34] is $1.6<\chi<3.3$.

The differential equation governing the motion of the structure during contact, obtained after summing the forces on the mass M (Fig. 2) is

$$M\frac{d^2x}{dt^2}+c\frac{d^\eta x}{dt^\eta}+kx-\kappa\delta^{\chi}=E\ \cos(\Omega t) \tag{2}$$

where x is the absolute displacement of the mass relative to a fixed reference, k and c are the linear elastic stiffness and viscous damping coefficient respectively. E and Ω are the excitation force amplitude and angular frequency respectively. $d^\eta x/dt^\eta$ is the fractional derivative in the Caputo sense, $0<\eta\le 1$ is the order. The Caputo fractional derivative is defined by

$$\frac{d^\eta x}{dt^\eta}=\frac{1}{\Gamma(n-\eta)}\int_0^t (t-\tau)^{n-\eta-1}\frac{d^n x(\tau)}{d\tau^n}\,d\tau \tag{3}$$

where $\Gamma(\cdot)$ is the Gamma function and $n-1<\eta\le n$, $n\in\mathbb{N}$.

By expanding the contact force non-linearity around a static point l_0 ($\delta=x-l_0$) using the Maclaurin power series expansion

$$\delta^{\chi}\approx(-l_0)^{\chi}(1-\chi l_0^{-1}x+1/2(\chi-1)\chi l_0^{-2}x^2-1/6(\chi-2)(\chi-1)\chi l_0^{-3}x^3)+\mathcal{O}(x^4). \tag{4}$$

In the presence of a crack, a downward shift of the resonance angular frequency ω_0 and a soft spring non-linear vibration behavior occurs [35]. This imposes that $\chi=3$ in Eq. (4). The dynamic equation in Eq. (2) then has the form

$$\frac{d^2x}{dt^2}+2\lambda\frac{d^\eta x}{dt^\eta}+\omega_0^2x+ax^2+bx^3=F\ \cos(\Omega t), \tag{5}$$

where $\lambda=c/2M$ is the damping ratio, $\omega_0=\sqrt{(k-3\kappa l_0^2)/M}$, the non-linear multipliers $a=3\kappa l_0/M$ and $b=-\kappa/M$. The modified normalized (using the mass) excitation force $F=E/M$. The constant term $\kappa l_0^3/M$ with no dependency on $x(t)$ has been neglected because it appears in the solution of the response as an offset that can be removed by high-pass filtering.

It can be noticed from Eq. (5) that the vibration response of the cracked structure has become non-linear due to the presence of the quadratic and cubic terms in the equation of motion. The cubic term gives the frequency-response curve of a soft non-linear