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Influence of surface energy on the non-linear pull-in instability of nano-switches

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ABSTRACT

This paper investigates the pull-in instability of a nano-switch under electrostatic and intermolecular Casimir forces. The analysis is based on the geometrically non-linear Euler–Bernoulli beam theory with consideration of the surface energy. Through differential quadrature method (DQM), the pull-in voltages of the nano-switch are obtained. Results show that the effect of surface energy and geometrically non-linear deformation on the pull-in voltage depends on the length, height and initial gap of the nano-switch. In addition, the effect of intermolecular Casimir force on the pull-in voltage weakens as the initial gap increases.

1. Introduction

Electrostatic nano-switches are important building blocks in nanoelectromechanical systems (NEMS). The pull-in instability in NEMS switches has attracted attentions of both research and industrial communities. Generally, a nano-switch is comprised of two electrodes separated by a dielectric medium. One is the fixed electrode which is modeled as a ground plane; another is the movable electrode which is modeled as a nano-beam. When a voltage is applied across the two electrodes, the movable electrode will be deformation due to the electrostatic force. Once this voltage increases beyond a critical value, the electrostatic force becomes larger than the corresponding restoring forces, resulting in the unstable collapsing of the movable electrode to the fixed electrode. This phenomenon is called as the pull-in instability. The critical voltage and deflection at this state are called as the pull-in voltage and pull-in deflection, which are important design parameters to describe the instability and eventually the switching between states.

Since the characteristic sizes of these NEMS switches are at nanoscale, the Casimir force or the van der Waals force can play an important role in the deflection and pull-in performance of nano-switches, depending on the gap between the two electrodes [1,2]. The Casimir force is known to be dominant when the gap between the electrodes is larger than 20 nm, and the van der Waals force becomes dominant when the gap between the electrodes is less than 20 nm [3].

Many studies of the influence of molecular interaction (Casimir and van der Waals force) on the deflection and pull-in performance of nano-structures have been carried out [3–10]. In particular, Buks and Roukes [4,5] investigated how the electrostatic and Casimir force determine the range of positional stability of electrostatically actuated devices by using experiments. Ramezani and Alasty [6] studied the pull-in instability of cantilever arrays with consideration of the Casimir force between the neighboring beams and electrostatic forces. Ramezani and Alasty [7] also investigated the effect of van der Waals force on the pull-in instability of cantilever arrays. Ramezani et al. [3] provided a distributed parameter model to investigate the effect of Casimir force or van der Waals force on the pull-in instability of cantilever switches. Guo and Zhao [9] investigated the stability of the electrostatic torsional NEMS actuators with consideration of Casimir force or van der Waals force. Taking the small scale effect into amount, Yang et al. [10] analyzed the pull-in instability of nano-switches under electrostatic force and Casimir force or van der Waals force, by using non-local elasticity theory. However, the above-mentioned works are limited to geometrically linear deformation only.

Electrostatically actuated nanostructures can suffer large deformation at some geometric configurations or under large electric force. The results of Abdel-Rahman et al. [11], Kuang and Chen [12], Tilmans and Legtenberg [13] and Hung and Senturia [14] all indicated that the geometrically non-linear deformation has significant effect on the pull-in behavior of analog-tuned electrostatic actuators. Batra et al. [15,16] used reduced-order models to study the pull-in instability of

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the electrostatically actuated micro-plates, with consideration of the von Karman type geometric non-linearity and Casimir force. Hsu [17] analyzed the eletromechanical behaviors of nano-actuators with considering geometrically non-linear deformation, by using differential quadrature method (DQM). Jia et al. [18] investigated the pull-in instability of micro-switches with considering geometric non-linearity deformation and the combined electrostatic and Casimir forces and axial residual stress. In their works, the non-linear governing differential equation is solved by using the differential quadrature method (DQM).

On the other hand, due to inherently large surface area to volume ratio of nanostructure, surface energy can play a crucial role in the pull-in behavior of NEMS switches. Ma et al. [19], Koochi et al. [20] and Yang et al. [21] studied the influence of surface energy on the pull-in instability of NEMS electrostatic switches, based on geometrically linear deformation. They found that the surface energy plays a significant role in the pull-in behavior of NEMS switches. Fu and Zhang [22] provided a modified continuum model of electrically actuated nanobeams with consideration of surface energy. However, the intermolecular forces (Casimir force or van der Waals force) are no taken into consideration.

In literature, the intermolecular forces, geometrically non-linear deformation and surface energy have been considered separately for the electrostatically actuated nanostructure. As described above, they all play significant roles in the pull-in behavior of NEMS switches. It is natural to take them into account simultaneously. Motivated by this fact, the present paper studies the effects of surface energy and geometrically non-linear deformation on the pull-in parameters of NEMS switches with consideration of Casimir force. The governing equations of NEMS switches are derived by using the principle of minimum potential energy. The equations are solved numerically through differential quadrature method (DQM). The proposed model is validated by comparing the results without considering surface energy with the previous analytical and experiments results. The effects of surface energy, geometrically non-linear deformation and Casimir force on the pull-in parameters of NEMS switches are investigated.

2. Theoretical formulations

Shown in Fig. 1(a) is a clamped–clamped nano-switch with including two components. One is the fixed electrode and the anther is the movable electrode with length L, width b, and thickness h. The two components are separated by a dielectric spacer with an initial gap g_0. In this paper, the gap is assumed to be larger than 20 nm such that the intermolecular forces between the electrodes are dominated by the Casimir force [1,2]

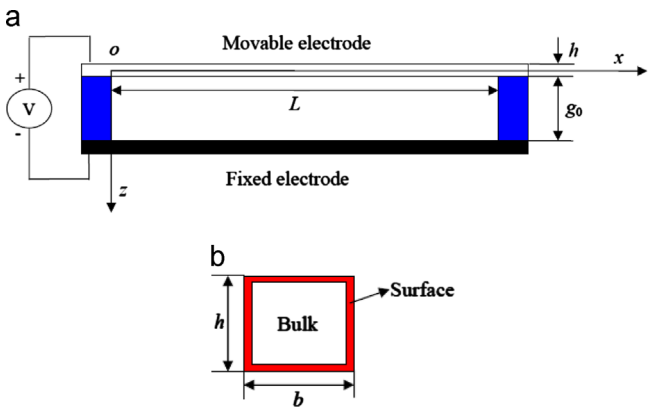

Fig. 1. (a) Beam model for a nano-switch and (b) a cross-section with surface layer.

$$F_c = \frac{\pi^2 \bar{h} c b}{240(g_0 - w)^4} \tag{1}$$

where $\bar{h} = 1.055 \times 10^{-34}$ J s is the Planck's constant divided by 2π, $c = 3 \times 10^8$ m/s is the speed of light. Consideration of fringing field correction, the electrostatic force per unit length is expressed as [23,24]

$$F_e = \frac{\varepsilon_0 b V^2}{2(g_0 - w)^2}\left(1 + 0.65\frac{g_0 - w}{b}\right) \tag{2}$$

where $\varepsilon_0 = 8.854 \times 10^{-12}$ C^2 N^{-1} m^{-1} is the permittivity of vacuum, w is deflection of the movable electrode. It is noted that the fringing field correction adopted here is first-order since it rather contains a correction term that accounts for the fringing fields from the top part of the cross section of the beam (part of the cross section not directly facing the bottom electrode).

The displacements of the nano-beam are $u = u_0 - z\,\partial w/\partial x$ and $w = w_0$, where u_0 and w_0 are the axial and transverse displacements of the point on the mid-plane ($z=0$). According to von Karman-type non-linear strain theory, the geometrical non-linear strain is

$$\varepsilon_x = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{3}$$

The strain energy of bulk of the nano-beam can be calculated from

$$\begin{aligned} U_b &= \int_V \int_0^{z_x} \sigma_x d\varepsilon_x dV = \frac{E}{2}\int_V \left(u_{0,x} - z w_{,xx} + \frac{1}{2}w_{,x}^2\right)^2 dV \\ &= \int_0^L \left[\frac{E}{2}bh\left(u_{0,x}^2 + \frac{1}{2}w_{,x}^2\right)^2 + \frac{bh^3 E}{24}(w_{,xx})^2\right] dx \end{aligned} \tag{4}$$

where E is Young's modulus.

Now, we calculate the energy of the surface. For a differential surface element with length dx, its length ds after deformation is $ds = \sqrt{(1+u_{,x})^2 + w_{,x}^2}dx$. Expanding ds in a binomial series and neglecting the higher-order terms, we obtain

$$ds = [1 + u_{,x} + \tfrac{1}{2}w_{,x}^2]dx \tag{5}$$

In the physics of solids, the energy of the molecules on the surface is larger than that of the molecules in the bulk of the material. The surface energy can be defined as the excess energy at the surface compared to the bulk. For one dimensional structure, surface energy can be expressed as [25]

$$\gamma = \gamma_0 + \tfrac{1}{2}E_s\varepsilon_x^2 \tag{6}$$

where γ_0 and E_s are, respectively, the residual surface energy and surface elastic modulus. The total surface energy can be calculated from $U_s = \int_S \gamma dS$, where S is the surface of the nano-beam. As a result, the total energy of the surface of the beam is

$$U_S = H\int_0^L \left[1 + \frac{1}{2}(w_{,x})^2\right]dx + \frac{EI^s}{2}\int_0^L w_{,xx}w_{,xx}dx + \frac{EA^s}{2}\int_0^L \left(u_{0,x} + \frac{1}{2}w_{,x}^2\right)^2 dx \tag{7}$$

where $H = 2\tau_0(b+h)$ and $EI^s = h^2 bE_s/2 + h^3E_s/6$ and $EA^s = 2(b+h)E_s$. The potential energy of Casimir force and electrostatic force is

$$W_q = -\int_0^L \int_0^w q dw dx \tag{8}$$

where

$$q = F_c + F_e$$