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On the coefficients of the interaction forces in a two-phase flow of a fluid infused with particles

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ABSTRACT

In this short paper we study the flow of a mixture of a fluid infused with particles in a channel. We use the classical mixture theory approach whereby constitutive relations are proposed for the stress tensor of each phase. For the interaction forces, the effect of different hindrance functions for the drag force is studied; moreover a generalized form of the expression for the coefficients of the interactions forces, also known as the hindrance functions, is suggested. For studying this two-component system numerically, a three-dimensional CFD solver based on OpenFOAM® has been developed. Applying this solver, a specific problem (blood flow) has been studied for which our numerical results and experimental data show good agreement.

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1. Introduction

From a continuum mechanics point of view, when describing the behavior of materials such as rubber, water, metals, or polymers, one can regard each of these as a single continuum. However, in many instances, we encounter ideal gas mixtures, fluid mixtures, bubbly liquids, alloys, composites, suspensions, fluidized particles, porous media and pneumatic systems that cannot be regarded as a single constituent continuum. It is more appropriate to assume that the material consists of more than one constituent. Another example of such a complex situation is a granular system which is a collection of a large number of discrete (solid) particles with interstices filled with a fluid, namely a liquid or a gas [24]; in general, the term 'granular materials' (solid particles) refer to particles with a size larger than 1 μm. Granular materials are unlike solids, since they conform to the shape of the vessel containing them, thereby exhibiting fluid-like behavior. However, they are also unlike fluids since they can be heaped. Many problems related to granular systems, whether occurring in nature or industry, have been the subject of extensive study. Some examples occurring in nature are sand dunes and snow avalanches. In many granular systems, the interstitial fluid plays a minor role in the bulk transport of materials and in such cases the system can be treated as a single component rather than multi-component system. However, in some granular systems, such as blood flow and coal feeder in a power plant (where the coal particles are transported to the combustion chamber by mixing them with oil or water), the mass of the interstitial fluid is comparable to that of the solid particles and the interactions between the fluid and the granular materials are significant; in such cases it is more appropriate to model the system as a multi-component one.

The flow behavior of fluid–solid mixtures in transport lines has been of interest in chemical processes for many years. In general, empirical relations that predict the flow rate and pressure drop in such processes have been developed for specific ranges of solids and fluid properties as well as for various geometries. A traditional way of modeling the behavior of fluids or suspensions that cannot be modeled by the classical Newtonian fluid model is by using non-Newtonian (or more accurately non-linear) models that are homogeneous. The theory of multiphase mixtures presents another avenue of research which can account for the inherent non-homogeneities in the problem. Modeling multi-component flows such as fluidized beds and slurries, quite often involve formulation of constitutive relations for quantities such as partial stress tensors and interaction forces. These constitutive relations inevitably introduce unknown material properties such as viscosities, normal stress coefficients, and thermal conductivities. These properties, in general, need to be measured experimentally, though in rare cases other theories, for example the statistical theory (for example, the kinetic theory of gases) may be used to obtain the form and dependence of these material coefficients on other measurable quantities. Many of the studies concerning

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fluid–solid flows typically employ one of the two continuum theories developed to describe such situations: averaging method or mixture theory (theory of interacting continua). In the *first* approach, namely the averaging method [28,29,23,20] point-wise equations of motion, valid for a single fluid or a single particle, are modified to account for the presence of the other components and the interactions between components. These equations are then averaged over time, some suitable volume which is large compared with a characteristic dimension (for example, particle spacing or the diameter of solid particles) but small compared to the dimensions of the whole system, or an ensemble. Terms which appear due to the process of averaging, which are not present in the equation being averaged, are usually interpreted as some form of interaction between the constituents. Constitutive relations to represent these interactive forces, as well as the stress tensors for each constituent, are required to make the theory complete. The *second* method of modeling multi-component systems is mixture theory. This theory, also known as the theory of interacting continua, based on modern continuum mechanics, can be applied to the above-mentioned problems. Truesdell [66] is the first to set up the mathematical theory of mixtures in which phenomena of diffusion, dissociation, combination, and chemical reaction in the broadest sense can be represented. Later, Truesdell [67] studied the mathematical theory of the diffusion of mass in a mixture using four different approaches, namely (i) the kinematical, leading to Fick's equation of diffusion, (ii) the hydrodynamical (Maxwell–Stefan equations of motion for the constituents in a mixture of fluids), (iii) the kinetic (Maxwell–Chapman–Enskog formulas in a mixture of dilute monatomic gases), and (iv) the thermodynamic approach, which is suitable for the diffusive flux in more general fluid mixtures. Kelly [33] presented one of the first attempts at a general theory for a reacting continuum. The mixture theory is in a sense, a homogenization approach in which each component is regarded as a single continuum and at each instant of time, every point in space is considered to be occupied by a particle belonging to each component of the mixture [68]. More detailed information, including an account of the historical development, is available in the review articles by Atkin and Craine [6,7], Bowen [11], and Bedford and Drumheller [10], and in the books by Truesdell [68], Samohyl [57], Rajagopal and Tao [49], Hutter and Johnk [25] and Schneider and Hutter [58].

In this paper we study the flow of a mixture of a fluid infused with particles in a channel. We use the classical mixture theory approach whereby constitutive relations are proposed for the stress tensor of each phase [38,39]. For the interaction forces, the effect of different hindrance functions for the drag force is studied; moreover a generalized form of the expression for the coefficients of the interactions forces, also known as the hindrance functions, is suggested. For studying this two-component system, a three-dimensional CFD solver based on OpenFOAM® is developed and the flow of such a mixture is studied in a micro-channel.

2. Governing equations

In this section, we provide a brief summary of the relevant and important equations in mixture theory. Let $\boldsymbol{X}_f$ and $\boldsymbol{X}_s$ denote the position of the bodies belonging to the interstitial fluid and that of the solid particles, in the reference configuration, i.e., prior to mixing. The motion of the two components can be represented as (see [11])

$$\boldsymbol{x}_f = \chi_f(\boldsymbol{X}_f, t), \quad \boldsymbol{x}_s = \chi_s(\boldsymbol{X}_s, t) \tag{1}$$

while the kinematical quantities associated with these motions are

$$\boldsymbol{v}_f = \frac{d_f \boldsymbol{x}_f}{dt}, \quad \boldsymbol{v}_s = \frac{d_s \boldsymbol{x}_s}{dt} \tag{2}$$

$$\boldsymbol{D}_f = \frac{1}{2}\left(\frac{\partial \boldsymbol{v}_f}{\partial \boldsymbol{x}_f} + \left(\frac{\partial \boldsymbol{v}_f}{\partial \boldsymbol{x}_f}\right)^T\right), \quad \boldsymbol{D}_s = \frac{1}{2}\left(\frac{\partial \boldsymbol{v}_s}{\partial \boldsymbol{x}_s} + \left(\frac{\partial \boldsymbol{v}_s}{\partial \boldsymbol{x}_s}\right)^T\right) \tag{3}$$

where $\boldsymbol{v}$ is the velocity field, $\boldsymbol{D}$ is the symmetric part of velocity gradient, and d_f/dt and d_s/dt denote differentiation with respect to time holding $\boldsymbol{X}_f$ and $\boldsymbol{X}_s$ fixed, respectively. Note that in the above equations the indices 'f' and 's' are not repeated indices; they stand for fluid and solid components, respectively. The bulk density field, ρ_f and ρ_s, for these two components are

$$\rho_f = (1-\phi)\rho_{f0}, \quad \rho_s = \phi\rho_{s0} \tag{4}$$

where ρ_{f0} and ρ_{s0} are the pure density of interstitial fluid and solid, in the reference configuration; ϕ is the volume fraction of solid, where $0 \le \phi < \phi_{\max} < 1$. The function ϕ is represented as a continuous function of position and time; in reality, ϕ is either one or zero at any position and at anytime, depending upon whether one is pointing to a solid particle or to the interstitial fluid at that position. That is, the real volume distribution has been averaged, in some sense, over the neighborhood of any given position. It should be mentioned that in practice ϕ is never equal to one; its maximum value, generally designated as the maximum packing fraction, depends on the shape, size, method of packing, etc.

In the absence of thermo-chemical and electromagnetic effects, the governing equations consist of the conservation of mass, linear momentum and angular momentum. The equations of conservation of mass in the Eulerian form are (see [6,7])

$$\frac{\partial \rho_f}{\partial t} + div(\rho_f \boldsymbol{v}_f) = 0 \tag{5a}$$

$$\frac{\partial \rho_s}{\partial t} + div(\rho_s \boldsymbol{v}_s) = 0 \tag{5b}$$

where $\partial/\partial t$ is the derivative with respect to time, *div* is the divergence operator, while the equations of balance of the linear momentum are written

$$\rho_f \frac{D^f \boldsymbol{v}_f}{Dt} = div(\boldsymbol{T}_f) + \rho_f \boldsymbol{b}_f + \boldsymbol{f}_I \tag{6a}$$

$$\rho_s \frac{D^s \boldsymbol{v}_s}{Dt} = div(\boldsymbol{T}_s) + \rho_s \boldsymbol{b}_s - \boldsymbol{f}_I \tag{6b}$$

where in general for any scalar β, $D^\alpha \beta/Dt = \partial\beta/\partial t + \boldsymbol{v}^\alpha \cdot \nabla\beta$, $\alpha = f, s$, and (for any vector $\boldsymbol{w}$), $D^\alpha \boldsymbol{w}/Dt = \partial \boldsymbol{w}/\partial t + (\nabla \boldsymbol{w})\boldsymbol{v}^\alpha$, $\mathbf{T}_f$ and $\mathbf{T}_s$ stand for the Cauchy stress tensors, $\mathbf{f}_I$ represents the interaction forces (exchange of momentum) between the components, and $\mathbf{b}_f$ and $\mathbf{b}_s$ refer to the body force. $\mathbf{T}_f$, $\mathbf{T}_s$ and $\mathbf{f}_I$ will be given by the constitutive equations. The balance of the angular momentum implies that, in the absence of couple stresses, the total Cauchy stress tensor is symmetric. Once the individual (partial) stress tensors are derived (or proposed), a mixture stress tensor can be defined as $\boldsymbol{T}_m = \boldsymbol{T}_1 + \boldsymbol{T}_2$ [21,22] where $\boldsymbol{T}_1 = (1-\phi)\mathrm{T}_f$ and $\boldsymbol{T}_2 = \boldsymbol{T}_s$ so that the mixture stress tensor reduces to that of the fluid as $\phi \to 0$ and to that of the solid particles as $\varepsilon \to 0$ [where $\varepsilon = (1-\phi)$]. $\boldsymbol{T}_2$ may also be written as $\boldsymbol{T}_2 = \phi\widehat{\boldsymbol{T}_s}$, where $\widehat{\boldsymbol{T}_s}$ may be thought of as representing the stress tensor in the reference configuration of the solid particles. Eqs. (5) and (6) represent the basic governing equations for the flow of a two-component mixture where there are no thermal, chemical, or electromagnetic effects. Clearly in most applications the thermal effects are important. In such cases, a complete thermodynamical theory is necessary, and in addition to the energy equations for the different components, one also needs to consider the effects of the second law of thermodynamics, i.e., the entropy inequality, often referred to as the Clausius–Duhem inequality. This is beyond the scope of the present study. Application of the Clausius–Duhem inequality, in general, would impose certain constraints on the material coefficients appearing in the constitutive relations (see [69]). A very