International Journal of Non-Linear Mechanics 80 (2016) 3–13

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Trapping vibratory energy of main linear structures by coupling light systems with geometrical and material non-linearities

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ARTICLE INFO

Article history:
Received 22 October 2014
Received in revised form
16 September 2015
Accepted 28 November 2015
Available online 24 December 2015

Keywords:
Non-linear energy sink
Hysteresis
Bouc–Wen
Passive control

ABSTRACT

Cubic potential and hysteresis behavior (Bouc–Wen type) of a non-linear energy sink are used to localize the vibratory energy of a linear structure. A general methodology is presented to deal with time evolutionary energy exchanges between two oscillators. Invariant manifold of the system and its stability borders are detected at fast time scale while traced equilibrium and singular points at slow time scale let us predict possible behaviors of the system during its pseudo-stationary regime(s). The paper is followed by an example that considers the Dahl model for representing the hysteresis behavior of the non-linear energy sink. All analytical developments and results are compared with those obtained by direct integration of system equations. Obtained analytical developments can be endowed for designing non-linear energy sink devices with hysteresis behavior to localize vibratory energy of main structures for the aim of passive control, energy harvesting and/or both of them.

1. Introduction

Quite intensive works have been carried out to present vibratory energy localization and passive control of main systems by non-linear energy sink (NES) devices with essentially cubic [1,2] or non-smooth non-linearities [3–10] for the aim of passive control, energy harvesting and/or for re-distribution of modal energy of main systems [11]. However in most of the above-mentioned researches, the main oscillator was supposed to be linear or to present smooth non-linearity with constant mass [12] or with time-dependent mass [13]. Some researchers consider non-smooth behavior of main structures during energy exchange process with a NES. In detail the following coupled systems have been analyzed: a main oscillator with piece-wise linear and also Dahl-type behavior [14] and a coupled non-smooth NES [7,15]; the main system with hysteresis behavior of Bouc–Wen type [16] and a NES with general non-linear potential function [17]; the main structure with single or several Saint-Venant elements [18] in parallel and a NES with cubic or general potential function [19,20]. In this paper we study time multi-scale energy exchanges between a main oscillator with linear behavior and another oscillator with dual non-linearities: a NES with smooth non-linear geometrical (cubic) and non-smooth hysteresis (Bouc–Wen) behaviors. The re-scaled form of system equations will be transferred to the center of mass and relative displacement; after injecting complex variables to the system and keeping first harmonics, the system behavior around 1:1 resonance will be analyzed at different scales of time. This multi-scale analysis will let us to predict invariant manifold and attraction points which can explain the passive control process of main structures or localization of their vibratory energy by NES devices with hysteresis responses. Organization of the paper is as it follows: mathematical representation of the system under consideration is presented in Section 2; analytical treatments of the system which includes change of variables, complexification of the system and applying truncated Galerkin's technique is discussed in Section 2.1. A general methodology for detecting behavior of two general coupled oscillators at different time scales is explained in Section 3; in Section 4 the same methodology is used to study time multi-scale behaviors of the considered coupled systems of this paper. As an example of hysteresis behavior of the NES, a Dahl model is chosen in Section 5; obtained results with direct numerical integrations of the system equations are compared with those obtained by analytical developments in Section 5.2. Finally, the paper is concluded in Section 6.

2. The model

We consider the academic model of the system under consideration which is depicted in Fig. 1: the main structure with the mass M, the linear stiffness k_1, damping c, displacement x_1, that is under external periodic force $\Gamma \sin(\Omega t)$. It is coupled to a NES with a very light mass $m = \epsilon M$ $(0 < \epsilon \ll 1)$, the damping λ and displacement x_2. The NES presents both non-linear geometrical and hysteresis behaviors. Its non-linear geometrical potential function reads $F(\alpha) = k_3 \alpha^3$ while its hysteresis behavior is supposed to be of Bouc–Wen type, $f(a, k_2, A, \beta, n, \gamma)$, with the following characteristics: x_3 is the internal variable of the

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http://dx.doi.org/10.1016/j.ijnonlinmec.2015.11.011
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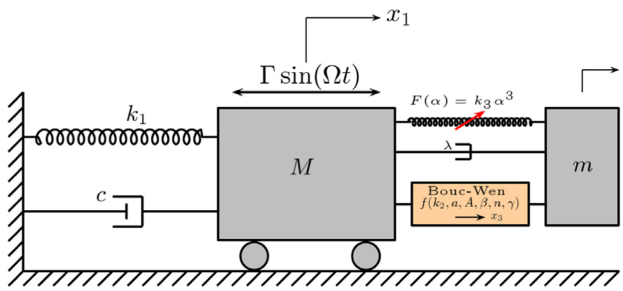

Fig. 1. A linear main structure coupled to a NES with cubic geometrical potential function ($F(\alpha)=k_3\alpha^3$) and a Bouc–Wen type hysteresis behavior ($f(k_2,a,A,\beta,n,\gamma)$).

hysteresis model, k_2 is the initial linear stiffness, a is the ratio of the post-yield (k_p) to initial stiffness i.e. $a=\frac{k_p}{k_2}$ and A, β, n and γ are parameters of the Bouc–Wen model that control hysteresis behavior. Governing system equations can be summarized as it follows:

$$\begin{cases} M\ddot{x}_1+c\dot{x}_1+k_1x_1+ak_2(x_1-x_2)+(1-a)k_2(x_1-x_3)+\lambda(\dot{x}_1-\dot{x}_2) \\ \quad +k_3(x_1-x_2)^3=\Gamma\sin(\Omega t) \\ m\ddot{x}_2+\lambda(\dot{x}_2-\dot{x}_1)+ak_2(x_2-x_1)+(1-a)k_2(x_3-x_1)+k_3(x_2-x_1)^3=0 \\ \dot{x}_3=A\dot{x}_2-\beta|\dot{x}_2||x_3|^{n-1}x_3-\gamma\dot{x}_2|x_3|^n \end{cases} \tag{1}$$

2.1. Re-scaling and complexification of system equations and applying Galerkin's technique

Let us shift the time t to the new domain as $T=\sqrt{\frac{k_1+k_2}{M}}t=\overline{\theta}t$. The system (1) reads $(x_1(t),x_2(t),x_3(t))\to(y_1(T),y_2(T),z(T))$:

$$\begin{cases} \ddot{y}_1+\epsilon\xi\dot{y}_1+\tilde{k}_1y_1+a\epsilon\tilde{k}_2(y_1-y_2)+(1-a)\epsilon\tilde{k}_2(y_1-z) \\ \quad +\epsilon\tilde{\lambda}(\dot{y}_1-\dot{y}_2)+\epsilon\tilde{k}_3(y_1-y_2)^3=\epsilon\tilde{f}_0\sin(\tilde{\omega}T) \\ \ddot{y}_2+\tilde{\lambda}(\dot{y}_2-\dot{y}_1)+a\tilde{k}_2(y_2-y_1)+(1-a)\tilde{k}_2(z-y_1)+\tilde{k}_3(y_2-y_1)^3=0 \\ \dot{z}=A\dot{y}_2-\beta|\dot{y}_2||z|^{n-1}z-\gamma\dot{y}_2|z|^n \end{cases} \tag{2}$$

where $\frac{k_2}{k_1+k_2}=\epsilon\tilde{k}_2$, $\frac{k_1}{k_1+k_2}=o(\epsilon^0)+o(\epsilon^1)=\tilde{k}_1=1-\epsilon\tilde{k}_2$, $k_3=\epsilon\tilde{k}_3$, $\frac{c}{\sqrt{M(k_1+k_2)}}=\epsilon\xi$, $\frac{\lambda}{\sqrt{M(k_1+k_2)}}=\epsilon\tilde{\lambda}$, $\frac{\Gamma}{\sqrt{k_1+k_2}}=\epsilon\tilde{f}_0$ and $\tilde{\omega}=\frac{\Omega}{\overline{\theta}}$. By shifting the system to the center of mass and relative displacement coordinates as $v(T)=y_1(T)+\epsilon y_2(T)$ and $w(T)=y_1(T)-y_2(T)$, the following equations can be obtained:

$$\begin{cases} \ddot{v}+\epsilon\xi\frac{\dot{v}+\epsilon\dot{w}}{1+\epsilon}+\tilde{k}_1\frac{v+\epsilon w}{1+\epsilon}=\epsilon\tilde{f}_0\sin(\tilde{\omega}T) \\ \ddot{w}+\epsilon\xi\frac{\dot{v}+\epsilon\dot{w}}{1+\epsilon}+\tilde{k}_1\frac{v+\epsilon w}{1+\epsilon}+a\tilde{k}_2(1+\epsilon)w+\tilde{\lambda}(1+\epsilon)\dot{w} \\ \quad +(1-a)\tilde{k}_2(1+\epsilon)\left(\frac{v+\epsilon w}{1+\epsilon}-z\right)+(1+\epsilon)\tilde{k}_3w^3=\epsilon\tilde{f}_0\sin(\tilde{\omega}T) \\ \dot{z}=A\frac{\dot{v}-\dot{w}}{1+\epsilon}-\beta\left|\frac{\dot{v}-\dot{w}}{1+\epsilon}\right||z|^{n-1}z-\gamma\left(\frac{\dot{v}-\dot{w}}{1+\epsilon}\right)|z|^n \end{cases} \tag{3}$$

The following complex variables of Manevitch [21] are applied to the system (3):

$$\begin{cases} \varphi_1e^{i\tilde{\omega}T}=\dot{v}+i\tilde{\omega}v \\ \varphi_2e^{i\tilde{\omega}T}=\dot{w}+i\tilde{\omega}w \\ \varphi_3e^{i\tilde{\omega}T}=\dot{z}+i\tilde{\omega}z \end{cases} \tag{4}$$

We endow Galerkin's technique by keeping first harmonics of the system and truncating higher ones [9,17]. This means that for a general function $ד(ב_1,ב_2,\ldots)$ we should evaluate the following integral:

$$\chi(ב_1,ב_2,\ldots)=\frac{\tilde{\omega}}{2\pi}\int_0^{2\pi/\tilde{\omega}}ד(ב_1,ב_2,\ldots)e^{-i\tilde{\omega}T}\,dT \tag{5}$$

Let us define the following variables and functions:

$$\varphi_j=N_je^{i\delta_j},\quad j=1,2,3 \tag{6}$$

$$s=\tilde{\omega}T+\delta_3\Rightarrow ds=\tilde{\omega}\,dT \tag{7}$$

$$N_1e^{i(\delta_1-\delta_3)}-N_2e^{i(\delta_2-\delta_3)}=P+iQ\quad(P,Q\in\mathbb{R}) \tag{8}$$

with

$$\begin{aligned} P&=N_1\cos(\delta_1-\delta_3)-N_2\cos(\delta_2-\delta_3) \\ Q&=N_1\sin(\delta_1-\delta_3)-N_2\sin(\delta_2-\delta_3) \end{aligned} \tag{9}$$

$$R=\sqrt{P^2+Q^2}=\sqrt{N_1^2+N_2^2-2N_1N_2\cos(\delta_1-\delta_2)} \tag{10}$$

$$\vartheta=\arctan\left(\frac{Q}{P}\right) \tag{11}$$

$$B=-\frac{\beta}{2\pi(1+\epsilon)}e^{i\delta_3}\left(\frac{N_3}{\tilde{\omega}}\right)^n \tag{12}$$

$$C=-\frac{\gamma}{1+\epsilon}\frac{1}{\pi}e^{i\delta_3}\left|\frac{N_3}{\tilde{\omega}}\right|^n \tag{13}$$

$$J_1=\int_0^{2\pi}|\cos(\vartheta+s)||\sin(s)|^{n-1}\sin(s)\cos(s)\,ds \tag{14}$$

$$J_2=\int_0^{2\pi}|\cos(\vartheta+s)||\sin(s)|^{n-1}\sin(s)\sin(s)\,ds \tag{15}$$

$$J_3=\int_0^{2\pi}\cos(\vartheta+s)|\sin(s)|^n\cos(s)\,ds \tag{16}$$

$$J_4=\int_0^{2\pi}\cos(\vartheta+s)|\sin(s)|^n\sin(s)\,ds \tag{17}$$

The system (3) by keeping its first harmonics, i.e. considering Eqs. (5)–(17), reads

$$\begin{cases} \dot{\varphi}_1-\frac{\tilde{\omega}}{2i}\varphi_1+\frac{\epsilon\xi}{2(1+\epsilon)}(\varphi_1+\epsilon\varphi_2)+\frac{\tilde{k}_1}{(1+\epsilon)(2i\tilde{\omega})}(\varphi_1+\epsilon\varphi_2)=-\frac{i}{2}\tilde{f}_0\epsilon \\ \dot{\varphi}_2-\frac{\tilde{\omega}}{2i}\varphi_2+\frac{\epsilon\xi}{2(1+\epsilon)}(\varphi_1+\epsilon\varphi_2)+\frac{\tilde{k}_1}{(1+\epsilon)(2i\tilde{\omega})}(\varphi_1+\epsilon\varphi_2)+\tilde{\lambda}(1+\epsilon)\frac{\varphi_2}{2} \\ \quad +\frac{a\tilde{k}_2}{2i\tilde{\omega}}(1+\epsilon)\varphi_2+(1-a)\frac{\tilde{k}_2}{2i\tilde{\omega}}(\varphi_1+\epsilon\varphi_2)-(1-a)\frac{\tilde{k}_2}{2i\tilde{\omega}}(1+\epsilon)\varphi_3 \\ \quad -\frac{i}{2}(1+\epsilon)\frac{3\tilde{k}_3}{4\tilde{\omega}^3}|\varphi_2|^2\varphi_2=-\frac{i}{2}\tilde{f}_0\epsilon \\ \frac{\varphi_3}{2}=\frac{A}{2(1+\epsilon)}(\varphi_1-\varphi_2)+BR(J_1-iJ_2)+CR(J_3-iJ_4) \end{cases} \tag{18}$$

In the next section we will use a time multi-scale method [22] and we will analyze the system behavior around 1:1 resonance by imposing $\tilde{\omega}=1+\sigma\epsilon$.

3. Time multiple scale behavior of the system: a general methodology

We present a general methodology to deal with time multiple scale behaviors of two coupled system: the main system which is attached to a NES. The summary of the method and its goals is listed here:

- fast and slow time scales i.e. $\tau_0=T$ and $\tau_1=\epsilon T$ are introduced.
- invariant of the system at fast time scale, namely τ_0-invariant, should be detected. This invariant corresponds to system behaviors at the infinity of fast time scale.