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Non-linear interactions in the flexible multi-body dynamics of cable-supported bridge cross-sections

Marco Lepidi [a,*], Vincenzo Gattulli [b]

[a] DICCA - Dipartimento di Ingegneria Civile, Chimica e Ambientale, Università di Genova, Via Montallegro 1, 16145 Genova, Italy
[b] DICEAA - Dipartimento di Ingegneria Civile, Edile-Architettura e Ambientale, Università dell'Aquila, Via Gronchi 18, 67100 L'Aquila, Italy

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ABSTRACT

An elastic section model is proposed to analyze some characteristic issues of the cable-supported bridge dynamics through an equivalent planar multi-body system. The quadratic non-linearities of the four-degree-of-freedom model essentially describe the geometric coupling which may strongly characterize the dynamic interactions of the bridge deck and a pair of identical suspension cables (hangers or stays). The linear modal solution shows that the flexural and torsional modes of the deck (*global modes*) typically co-exist with symmetric or anti-symmetric modes of the cables (*local modes*). The combinations of parameters which realize remarkable 2:1:1 internal resonance conditions among one of the global modes (with higher natural frequency) and two local modes (with lower and close natural frequencies) are obtained by virtue of a multiparameter perturbation method. The non-linear response of the resonant systems shows that the global deck motion – directly forced at primary resonance by an external harmonic load – can parametrically excite the local cable motion, when the deck vibration amplitude overcomes the critical value at which a period-doubling bifurcation occurs. The relevant effects of both viscous damping and internal detuning on the instability boundaries are parametrically investigated. All the internal resonance conditions as well as the critical vibration amplitudes are expressed as an explicit, though asymptotically approximate, function of the structural parameters.

1. Introduction

Leveraging on the most recent developments in the structural design, industrial technology and architectural style, cable-supported bridges are encountering a growing worldwide success as functional, economic and elegant solutions to cover increasingly larger spans. As inevitable drawback, the simultaneous optimization of the geometric shape, material usage and visual pleasantness tends to exalt the inherent slenderness and flexibility of many structural schemes based on the cable-beam collaboration (Fig. 1a). These exalted features, combined with limited damping resources and associated with rising performance demands, expose modern suspension or cable-stayed bridges and footbridges to the severe risk of developing important dynamic phenomena. Unacceptable vibration amplitudes for the bridge safety or serviceability may be caused, for instance, by aeroelastic instabilities under the wind action, or dynamic bifurcations under the direct or parametric excitation of pedestrian or vehicular traffic.

In this challenging framework, continuous research efforts have been focused over the last decades on improving the accuracy and reliability of the analytical and numerical models describing the cable-supported bridge dynamics. Ever since, in the early nineties of the past century [1,2], the cable vibrations were clearly recognized to influence the full-bridge response in a non-negligible way, a leading branch in this research field is concerning the linear and non-linear interactions among *global modes*, dominated by the three-dimensional motion of the bridge deck, and *local modes*, dominated by the transversal motion of the stay cables or hangers (Fig. 1b and c). As primary consequence, the traditional models, coarsely describing the cables as transversely-motionless tendon elements with Ernst equivalent elastic modulus [3,4], have been progressively abandoned or confined to preliminary technical applications. Encouraged also by the increasing availability of computational resources, a variety of continuous and discrete formulations have been proposed for their replacement [5–12], in the continuous attempt to better account for the local cable dynamics within the high-dimensional numerical models of complex cable structures.

Together with the refinement of full-bridge models, a parallel research line has been oriented to clarify the crucial role played by the three-dimensional deck motion in the cable excitation mechanisms. In

* Corresponding author.
E-mail address: marco.lepidi@unige.it (M. Lepidi).

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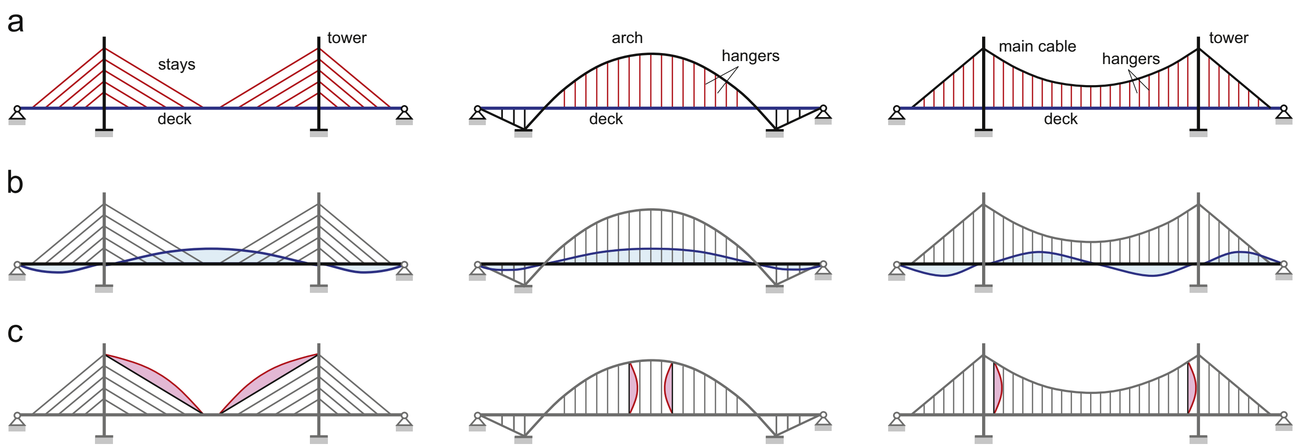

Fig. 1. Cable-stayed bridge (left), through-arch bridge (centre), suspended bridge (right): qualitative examples of (a) cable–beam collaboration schemes, (b) deck vibration shape in global modes, and (c) cable vibration shapes in local modes.

this respect, a number of low-dimensional analytical models have been proposed to parametrically investigate a single or a few specific phenomena. In particular, different monodimensional continuum models have been formulated to describe paradigmatic simple systems, typically composed of a non-linear Irvine cable joined with a linear simply-supported beam or cantilever [13–17]. The latest enrichments of these models include up to four cables sustaining a multi-span beam deck [18]. On the one hand, as primary research outcomes in the linear field, significant frequency veering and modal hybridization phenomena have been recognized [12,14]. On the other hand, as remarkable findings in the rich non-linear scenario, some aspects of the well-known mechanism of cable parametric excitation have been enlightened [13,19,20], new autoparametric sources of high-amplitude local vibrations have been identified [20,21] and transitions to chaotic dynamics have been described [16,17,22]. According to this well-established literature, high-amplitude cable oscillations can be determined by either linear or non-linear excitation mechanisms, generally regulated by commensurable frequencies of global and local modes. Consequently, global–local interactions have been invoked as physical explanation for the frequent reports of unacceptable levels of cable vibration in real long-span bridges [23,24]. From the engineering viewpoint, the matter tends to increase in relevance for the newest structural realizations. Indeed, the modern design trend is bent on reducing the bridge vulnerability to wind actions by further streamlining the already slender cross-sections of the deck, whereas the increment of the total number of hangers or stays is recommended against the static stress localization in the suspension system (main cables, arch, towers). Consequently, recently-built cable-stayed and suspension bridges often possess a highly-dense spectrum with several global and local modes concentrated in the low-frequency range, in which the internal resonances given by integer frequency ratios tend to become practically unavoidable. Finally, groups of two or more local modes with identical or close frequencies are the natural consequence of the ubiquitous symmetry properties in the bridge geometries.

Based on this background, the present paper is focused on the parametric analysis of the non-linear interactions among global and local modes in cable-supported bridges, with a threefold aim: first, to explicitly identify which combinations of the mechanical design parameters may determine the occurrence of internal sub- or super-harmonic resonances among two or more linear frequencies and, therefore, favour the energy transfer from global to local modes; second, to define a stability chart in the parameter space through the analytical prediction of the bifurcation phenomena governing the cable excitation mechanism; finally, to quantitatively estimate the post-critical vibration amplitudes as a closed-form function of the involved parameters. Although some of these issues can be analytically, numerically and experimentally studied by means of synthetic continuum models [20,21], their mathematical complexities inevitably limit the actual possibility to achieve consistent but simple parametric solutions, suitable for immediate technical applications as, for instance, the vibration mitigation. Moreover, the largest majority of existing continuum models are planar, and thus neglect a significant part of the complex three-dimensional bridge behaviour, including for instance the flexural–torsional coupling in the deck motion.

A proper balance between the competing requirements of synthesis and representativeness may be represented by multi-body mechanical systems. According to this leading idea, the authors have recently proposed an elastic section model to describe the section dynamics of suspension and cable-stayed bridges through an equivalent flexible multi-body system [25]. As original key feature, the model accounts for the transversal motion of the hangers or stays, which – even in the most refined deformable section models – are usually treated as equivalent linear or non-linear springs, whose effects on the deck dynamics are limited to mono-lateral stiffening contributions [26–30]. Moving beyond the analyses of the linearized dynamics [25] and aerodynamics [31], the multi-body model formulation is here enriched to account for the non-linear dynamics of cable-supported bridges (Section 2). All the objective results are sought for in an analytical, although asymptotically approximate, form. Indeed, a multiparameter perturbation technique is employed to solve the linear modal problem (Section 3) and the Multiple Scale Method is adopted to describe the forced non-linear response (Section 4). Focus is made on the high-amplitude, low-frequency oscillations of the cables, parametrically excited by the small-amplitude, high-frequency oscillations of either the flexural (Section 4.2) or the torsional deck motion (Section 4.3). The latter case, which to the authors' knowledge has never been studied by means of analytical section models, is peculiarly featured by the linear and non-linear coupling, of geometric nature, of three (namely one global, two local) modes. Concluding remarks are finally pointed out.

2. Multi-body section model

The non-linear cable–deck interactions in cable-stayed and suspended bridges may involve the vertical, lateral or torsional deck motion and the dominant transversal motion of one or more resonant cables. A planar multi-body model composed of a *principal subsystem* (*SP*), consisting of a rectangular rigid body, and two *secondary subsystems* (SS_1 and SS_2), consisting of a pair of identical point bodies, is adopted to synthetically describe the bridge dynamics in the cross-section plane (Fig. 2a). From the structural viewpoint, the model arrangement may be intended as a simplified geometric representation