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Multi-modes approach to modelling of vortex-induced vibration

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ABSTRACT

In this work the fluid–structure interactions are considered by investigating a straight but slender pipe interacting with uniform water flow. Two configurations are studied, namely vertically and horizontally positioned pipes, which are modelled as an Euler–Bernoulli beam with flexural stiffness. Both pretension and length-wise mass distribution are considered. The structure is assumed to be moving only in the direction normal to flow (cross-flow motion) hence its in-line motion is neglected. The external fluid force acting on the structure is the result of the action of sectional vortex-induced drag and lift forces. Only mean drag force is considered, with time varying lift force modelled using a non-linear oscillator equation of the Van der Pol type. The obtained coupled system of non-linear partial differential equations is simplified employing Galerkin-type discretisation. The resulting ordinary differential equations are solved numerically providing multi-mode approximations of cross-flow displacement and non-dimensional lift coefficient. The comparison between the responses of vertical and horizontal structures shows that, as expected, due to a balancing between pretension and weight, in general a higher amplitude of vibration is observed for the vertical configuration than in the same location along the pipe for the horizontal configuration in the lower part of the structure. However, lower amplitudes are obtained in the upper part of the pipe. The horizontal configuration solutions are identical in symmetrical locations along the pipe due to constant pretension. The influence of the wake equation coefficients and the fluid force coefficients on the response amplitudes has been also considered together with the length of the pipe and pretension level, and the appropriate response curves are included. Finally, for the higher mode approximations it has been shown that the vibrations level at lower frequencies is predicted reasonably well by retaining only a small subset of modes.

1. Introduction

Slender marine structures such as piles, offshore risers, umbilicals, and others are often very sensitive to excitation induced by vortex shedding, which results in vibrations that in certain combinations of waves and current develop into a resonance phenomenon known as lock-in. This kind of vibrations can be destructive and may lead to structural collapse. Several decades of research have been devoted to exploring the complex nature of vortex-induced vibrations (VIVs), from purely fundamental studies of the phenomenon to attempts to develop a reliable toolkit that would allow predicting VIVs of marine structures with a desired precision. It should be noted that VIVs can also develop in air, in particular in cable bridges, chimneys and tall buildings [1,2] but the current study is focussed on this phenomenon in the water.

The need of industry for an effective solution in this area led to the development of a number of software packages, such as SHEAR7 [3] or OrcaFlex [4], that utilise semi-empirical approaches to predict riser VIVs. One of the critical requirements for such tools is an acceptable simulation time. These tools normally use a coupled model of the fluid forces and the structural dynamics to simulate VIVs of a riser. The fluid force is usually modelled using semi-empirical equations such as Morison's equation. As a result, these tools are fast to run and they offer a cheaper and more practical alternative to carrying out physical experiments. However, the obvious drawbacks in this case are their limited accuracy and range of applicability.

When computational fluid dynamics became a more accessible way to model VIVs due to the increase of computational capabilities, a number of new modelling approaches were proposed, such as investigations and attempts to study the behaviour of long flexible circular cylinders and full-scale riser systems with the so-called strip theory [5–8], which allows the observation of a multi-mode response of a flexible riser while implementing high aspect ratio 3D meshes [9]. Although this was definitely a step forward,

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the computation of a single CFD simulation still takes a significant amount of time for this approach to be effectively integrated and used by the industry. Some attempts of partial integrations have been made by OrcaFlex [4] by introducing the Vortex Tracking model, based on the underlying physical equations of boundary layer theory and the Navier–Stokes equations. However, there are some benefits in using full-scale CFD as a tool to calibrate existing semi-empirical reduced-order models, which will result in creation of a better, less computationally demanding models for VIVs prediction. Experimental studies on the force distribution along the axis of a flexible circular cylinder undergoing multi-mode VIVs have also been carried out, for example by Chaplin et al. [10] and Huera Huarte et al. [11], who in 2006 combined experimental data with the finite element method model of the riser. Recently, Franzini et al. [12,13] obtained and used experimental data for determining hydrodynamic coefficients in the developed reduced order model of flexible cylinder subjected to prescribed top motions. Similar problem was considered by Mazzilli et al. [14] and non-linear modes based on Bessel functions were developed. These results could be also useful for the reduced-order model calibrations.

The analytical approach to VIVs is represented by a wide range of models, with some of them incorporating a Van der Pol type equation as the governing equation for the fluid force acting on the structure [15,16]. Since then, many modifications have been made to this particular approach, and the model itself became known as the wake oscillator model. The simplest wake oscillator model describes the interactions between a fluid flow and a rigid cylinder capable of moving in cross-flow direction only, with the system usually described by two coupled ordinary differential equations. One of the equations is the equation of motion of the structure, and the second equation is a semi-empirical description of the lift force generated by the passing fluid, a non-linear self-excited fluid oscillator. In 1997 Balasubramanian and Skop [17] proposed the new model with two terms for the lift force. In previous models, a Van der Pol equation was employed as the governing equation for the entire lift force on the structure. The model introduced in [17] included a Van der Pol equation driven by the local transverse motion of the structure as a governing equation for one component of the fluctuating lift force and a so-called stall term which is linearly proportional to the local transverse velocity of the structure. Because of the stall term, an asymptotic, self-limiting structural response at zero structural damping was captured.

One of the most notable analyses in this class of low dimensional models in terms of the fundamental behaviour was made by Facchinetti et al. [18], where the transverse vibrations of one degree-of-freedom structures in stationary uniform flow were investigated. The analysis was undertaken by first estimating the values of all parameters based on the available experimental data on forced vortex shedding, and then solving numerically the fully coupled system. These results have been systematically compared with experimental data from the literature such as oscillation amplitude at lock-in, extension of lock-in and effective added mass. It was found that the acceleration coupling provided the best match for the experimental results qualitatively and, in some aspects, quantitatively. Later, Violette et al. [19] performed comparisons of the Facchinetti wake oscillator model of slender cylinder undergoing VIVs with DNS and experiments.

A coupled model incorporating two degrees-of-freedom motion of a rigid cylinder excited by harmonic fluid forces was proposed by Wang et al. [20] in 2003, who also considered an extension of this approach towards a slender structure by using Euler–Bernoulli beam theory. This work was later modified by Ge et al. [21], incorporating the Facchinetti wake oscillator [18]. In 2007 Furnes and Sorensen [22] developed a 3D time domain model called VIVITAS [22] with coupled non-linear oscillators representing in-line and cross-flow fluid force components. Some empirical coefficients from the Facchinetti model [18] were tuned against the Marintek data [22]. The effect of variable tension on vortex induced vibration of vertical riser was studied in [23] where a finite element model of the structure was coupled to a wake-oscillator model of the Van der Pol type. Overall, the wake oscillator models are not computationally demanding and hence can be implemented in various software packages for the design and analysis of offshore marine systems, and for instance, the Iwan and Blevins model [24] is already included in OrcaFlex [4].

Further investigations on the semi-empirical approach were presented in the work by Keber and Wiercigroch [25]. They investigated the effect of a weak structural non-linearity on the dynamical behaviour of a vertical offshore riser undergoing VIVs. As offshore risers experience additional excitation from the flow of the fluid inside the pipe, this work considered the influence of internal flow and non-linear coupling of axial and bending displacements. It was shown that the structural non-linearity has a stiffening effect on the oscillation of the riser, which becomes more pronounced when the internal flow is incorporated into the model. Earlier Mazzilli et al. have conducted non-linear modal analysis of axially loaded beams in [26] exploring effects of structural non-linearities. In 2009 Srinil et al. [27] proposed a reduced-order model of the VIVs of a catenary riser, with a mathematical description of a multi-mode response of a slender structure combined with a wake oscillator model as the fluid force component; however, only very limited multi-mode analysis was presented. Catenary risers were also studied by Mazzilli and Sanches [28] using the non-linear normal mode approach where the fluid forces were modelled using the phenomenological model from [1]. The same approach was later used for straight risers modelling in [29] where the fluid forces were described by Iwan–Blevins phenomenological model.

Overall, although a number of models describing interactions between the fluid flow and flexible structures have been developed and published, the obtained numerical results are limited and there is still a need for proper multi-mode analysis. Several of the available models only look at the horizontal configuration of the structure, where the pretension is acting in the direction perpendicular to the distributed weight of the structure. The weight distribution becomes important in the vertical configurations, where these forces are acting along the same axis, and therefore this paper will look at the comparison of the multi-mode responses of vertical and horizontal configurations of a pipe.

The rest of the paper is organised as follows. In the next section the mathematical model of the riser system is explained, which is followed by a description of the proposed reduction procedure in Section 3. The responses are analysed in Section 4, where the dependence of the solution on key system parameters is shown and influence of the number of modes retained in the model on the system response is considered. Finally, some concluding remarks are provided in Section 5.

2. Description of the model

The adopted model for the fluid–structure interactions considered here consists of a straight but slender pipe interacting with uniform flow. Two configurations of the pipe are studied, i.e. vertically and horizontally positioned structures as shown in Fig. 1. The pipes are modelled as an Euler–Bernoulli beam with flexural stiffness. Structural damping can be neglected in the first instance as damping of risers is dominated by hydrodynamic forces and internal damping of the pipe is considered to be very low [30]. The pretension T is applied, and the length-wise mass distribution is included. The structure is assumed to be moving only in the