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Harmonic wavelets based response evolutionary power spectrum determination of linear and non-linear oscillators with fractional derivative elements

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ABSTRACT

A harmonic wavelets based approximate analytical technique for determining the response evolutionary power spectrum of linear and non-linear (time-variant) oscillators endowed with fractional derivative elements is developed. Specifically, time- and frequency-dependent harmonic wavelets based frequency response functions are defined based on the localization properties of harmonic wavelets. This leads to a closed form harmonic wavelets based excitation-response relationship which can be viewed as a natural generalization of the celebrated Wiener–Khinchin spectral relationship of the linear stationary random vibration theory to account for fully non-stationary in time and frequency stochastic processes. Further, relying on the orthogonality properties of harmonic wavelets an extension via statistical linearization of the excitation-response relationship for the case of non-linear systems is developed. This involves the novel concept of determining optimal equivalent linear elements which are both time- and frequency-dependent. Several linear and non-linear oscillators with fractional derivative elements are studied as numerical examples. Comparisons with pertinent Monte Carlo simulations demonstrate the reliability of the technique.

1. Introduction

Engineering structural systems are often subject to extreme events and excitations such as seismic motions, winds, hurricanes, ocean waves, blasts and impact loads which inherently possess the attribute of evolution in time. Further, most of these excitations exhibit not only time-varying intensities, but also time-varying frequency contents. Thus, it can be readily seen that there is a need for developing joint time–frequency analysis techniques for capturing accurately the system/structure behavior (e.g. [1–3]). Furthermore, in the field of stochastic structural dynamics (e.g. [4]) the concept of power spectrum is indispensable for characterizing and quantifying uncertainties prevalent in complex engineering systems. These uncertainties are mainly associated with excitations, and with structural dynamic responses. Clearly, there is a need to translate the aforementioned uncertainties into engineering load models and to develop response determination techniques, so that structural systems are efficiently designed, monitored, and maintained.

In this regard, research efforts have focused in recent years on utilizing wavelets for evolutionary power spectrum (EPS) estimation based on available process realizations (e.g. [5–7]). Nevertheless, note that unless a rigorous mathematical model exists for representing non-stationary stochastic processes via wavelets, questions are raised concerning any kind of wavelet-based time-dependent spectral analysis (e.g. see [8] for a related discussion). In this regard, Spanos and Kougioumtzoglou [9] developed a harmonic wavelets based statistical linearization technique for determining the non-linear system response EPS based on a rigorous wavelet-based representation of non-stationary stochastic processes [10]. Further, Kougioumtzoglou [11] developed an approximate analytical technique for determining the non-linear system response EPS based on the aforementioned theoretical framework of locally stationary processes and on the orthogonality properties of harmonic wavelets. This technique can be viewed as an extension and generalization of a widely used spectral relationship in stationary non-linear random vibration theory (see [12] and references therein) to account for non-stationary processes of arbitrary EPS.

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Furthermore, since the pioneering work by Gemant [13] and Bosworth [14], who first proposed fractional derivatives modeling for the constitutive behavior of viscoelastic media (see also [48–50] for some additional early contributions), fractional calculus has been successfully applied in diverse fields of physics and engineering such as viscoelasticity and rheology, control theory as well as biophysics and bioengineering (e.g. see [15] for a broad perspective). In particular, applications of fractional derivatives in structural engineering for vibration control or seismic isolation purposes include modeling of the restoring force of structural systems equipped with viscoelastic dampers (e.g. [16–19]). In this regard, theoretical developments have been found in very good agreement with experimental results (e.g. [20]).

Focusing on the stochastic response determination of linear and non-linear oscillators endowed with fractional derivative elements several both time and frequency domain numerical and approximate analytical techniques have been developed (e.g. [21–29]). Note that although frequency domain approaches are more efficient computationally than time domain simulation schemes, they are restricted to the stationary case only. Further, to the best of the authors' knowledge limited results, if any, exist in the context of a stochastic joint time–frequency response analysis of structural systems endowed with fractional derivative elements.

In this regard, in this paper a harmonic wavelets based approximate analytical technique is developed for determining the response EPS of linear and non-linear (time-variant) structural systems endowed with fractional derivative elements. Specifically, based on the localization properties of harmonic wavelets a frequency- and time-bands dependent excitation-response relationship is derived which can be viewed as a generalization of the celebrated Wiener–Khinchin spectral relationship of the linear stationary random vibration theory. In this manner, a joint time–frequency response analysis is achieved. Further, relying on the concept of defining both time- and frequency-dependent optimal equivalent linear elements, the aforementioned input–output relationship is extended via statistical linearization to account for non-linear systems. Overall, the technique can be construed as a generalization of the concepts/results obtained by Spanos and Kougioumtzoglou [9] and Kougioumtzoglou and Spanos [30] to account for systems with fractional derivative elements.

2. Mathematical formulation

2.1. Harmonic wavelets elements

In this section the basic properties of generalized harmonic wavelets (GHWs) are reviewed. In this regard, the family of GHWs (e.g. [31]) utilizes two parameters (m,n) for the definition of the bandwidth at each scale. The main advantage of this family relates to the decoupling of the time–frequency resolution achieved at each scale from the value of the central frequency; this is not the case with other commonly used wavelet bases such as the Morlet and other families.

Further, GHWs have a box-shaped frequency spectrum, whereas a wavelet of (m,n) scale and (k) position in time attains a representation in the frequency domain of the form

$$\Psi^{G}_{(m,n),k}(\omega)=\begin{cases}\frac{1}{(n-m)\Delta\omega}\exp\left(-i\omega\frac{kT_0}{n-m}\right), & m\Delta\omega\le\omega\le n\Delta\omega\\ 0, & \text{otherwise}\end{cases}, \tag{1}$$

where m, n and k are considered to be positive integers and

$$\Delta\omega=\frac{2\pi}{T_0}, \tag{2}$$

where T_0 is the total time duration of the signal under consideration. A collection of harmonic wavelets of the form of Eq. (1) spanning adjacent non-overlapping intervals at different scales along the frequency axis is shown schematically in Fig. 1. The inverse Fourier transform of Eq. (1) gives the time-domain representation of the wavelet which is equal to

$$\psi^{G}_{(m,n),k}(t)=\frac{\exp\left(in\Delta\omega\left(t-\frac{kT_0}{n-m}\right)\right)-\exp\left(im\Delta\omega\left(t-\frac{kT_0}{n-m}\right)\right)}{i(n-m)\Delta\omega\left(t-\frac{kT_0}{n-m}\right)}. \tag{3}$$

In Fig. 2, an example of the generalized harmonic wavelet (GHW) of Eq. (3) with parameters values $m=5, n=10, k=\ 2, \Delta\omega=1.7241\ \text{rad/s}, T_0=18.9\text{s}$ is plotted. Further, the continuous generalized harmonic wavelet transform (GHWT) is defined as

$$W^{G}_{(m,n),k}[f(t)]=\frac{n-m}{T_0}\int_{-\infty}^{\infty}f(t)\overline{\psi^{G}_{(m,n),k}}(t)dt, \tag{4}$$

and projects $f(t)$ on this wavelet basis; the bar over a symbol represents complex conjugation. Furthermore, perfect reconstruction of the original signal $f(t)$ can be achieved according to the equation

$$f(t)=\sum_{m,n}\sum_{k}\left(W^{G}_{(m,n),k}[f(t)]\psi^{G}_{(m,n),k}(t)+\overline{W^{G}_{(m,n),k}[f(t)]\psi^{G}_{(m,n),k}(t)}\right), \tag{5}$$

where $f(t)$ is assumed to be a zero-mean signal.

It is noted that Eq. (5) represents a GHWs based representation of deterministic functions. It can be readily understood that a mathematically rigorous wavelets based representation of stochastic processes is required to perform any kind of joint time–frequency analysis in a stochastic sense; see also [8] for a relevant

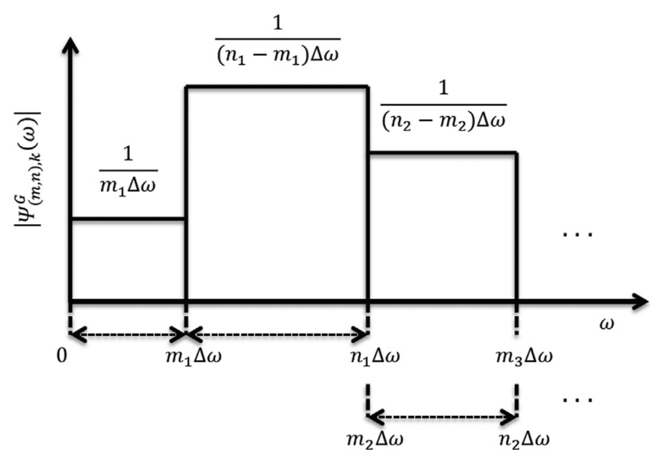

Fig. 1. A generalized harmonic wavelets basis example spanning non-overlapping intervals of arbitrary bandwidths in the frequency domain.

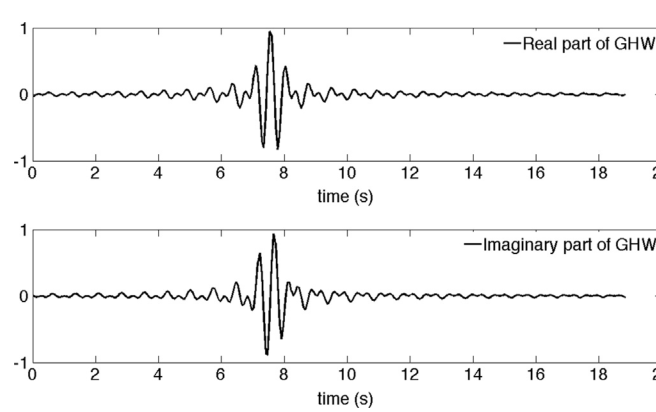

Fig. 2. Real and imaginary parts of a generalized harmonic wavelet (GHW) in the time domain with parameters values $m=5, n=10, k=2, \Delta\omega=1.7241\ \text{rad/s}$, $T_0=18.9\text{s}$.