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Continuous transition between traveling mass and traveling oscillator using mixed variables

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ABSTRACT

The interaction between cars or trains and bridges has been often described by means of a simplified model consisting of a beam loaded by a traveling mass, or by a traveling oscillator.

Among others, two aspects are essential when dealing with masses traveling along flexible vibrating supports: (i) a complete relative kinematics; and (ii) a continuous transition between a traveling mass, rigidly coupled, and a traveling oscillator, elastically coupled with the support.

The kinematics is governed by normal and tangential components—with respect to the curved trajectory—of the acceleration. However in literature these parts are oriented with reference to the undeformed beam configuration. This model is improved here by a non-linear second-order enriched contribution.

The transition between a traveling oscillator and a traveling mass is governed by the stiffness k of the elastic or viscoelastic coupling which, in the latter case (i.e. rigid coupling), has to tend towards infinity.

However, very large stiffness values cause high frequencies and significant problems are mentioned in the literature in order to establish numerically stable and reliable results and in order to realize a continuous evolution between absolute and relative formulations.

By using mixed state variables, generalized displacements and coupling forces, the contribution from the stiffness changes from k to its inverse $1/k$, the coupling force itself becomes a member of the solution-space and the problems, which have been mentioned in the literature, disappear. As a matter of fact, the coupling force can also take into account a viscoelastic contribution; moreover, a larger number of traveling oscillators can be considered, too.

Finally, for a periodic sequence of moving oscillators the dynamic stability is treated in the time-domain along several periods, as well as in the spectral domain, by using Floquet's theorem.

1. Introduction

The body of literature devoted to traveling oscillators is large. State-of-the-art overviews are available from Ouyang [1] and Au et al. [2]. The classical treatment concentrates on finding the critical constant velocity which leads to a continuous increase of deformations if a sequence of masses crosses the beam/bridge.

Well-known studies on this dynamic stability problem have been presented by Bolotin [3], Fryba [4], Luongo [5–8] and Piccardo and his coworkers [9–14].

In order to avoid dynamic instability, the use of piezoelectric actuators can be effective: some applications to beams and plates are shown in [15–18].

A group of recently published papers presents a discussion whether absolute or relative formulations should be used [19]; deals with the equivalence of the moving mass and moving oscillator problems [20,21]; and gives attention to the dynamic stability if a sequence of oscillators crosses the supporting structure, which has been already deformed by the foregoing oscillators [22].

Another group of papers [23,24] deals with more sophisticated models for both, bridge and vehicles, which are represented as an assembly of rigid bodies, springs and dampers. Sometimes, in addition, the bridge is modeled as a continuous in-plane curved beam or the wheel-rail interaction is of special concern: see, for instance [25–30].

Plates and beams on generalized foundations subjected to moving loads are treated in several papers like [31–33] including

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tensionless foundations and special models for finite and infinite soil domains using Boundary Elements, Infinite elements or the Scaled Boundary Finite Element Method.

The analysis and control of bridges with traveling masses has been studied, too, during earthquakes: several papers are listed in the above-mentioned review [1].

Finally, local and global changes of the supporting structure due to abrupt changes in the bridge-railway interface or due to separation and impact-reattachment have been treated in [34–36].

The first aspect of this paper concerns the introduction of mixed state variables, generalized displacements and coupling forces, for the description of the traveling oscillator. In doing so, a continuous transition between a traveling mass and a traveling oscillator can be established without incurring numerical problems due to high frequencies caused by stiffness coefficients tending towards infinity. The key idea behind this new approach is to change from the stiffness k to its inverse $1/k$; in doing so, the coupling force automatically appears as an additional member of the solution-space.

However, to prepare a common basis for the description of the deformations and the total acceleration of the oscillator, Section 2 describes in a detailed way the model of a circle-like beam in a horizontal plane. Thus, the straight beam case is rigorously recovered when the curvature radius R tends toward infinity; practically, it is included in the presented model when this radius is significantly increased.

In Section 3 the kinematic model of the traveling mass is presented and discussed. Section 4 is devoted to the second innovative aspect of this paper: a consistent representation of the normal and tangential acceleration with respect to the curved trajectory of the traveling mass or oscillator. These accelerations are part of the second total derivative $\mathrm{d}^2\mathbf{x}/\mathrm{d}t^2$ of the position vector $\mathbf{x}$ of the traveling coupling point: up to now, the classical formulation shows that the normal acceleration is applied along the vertical direction (which is a first-order approximation), and not along the normal to the supporting curve, which is bent by strains, and constitutes the actual trajectory traveled by the coupling point. Here, a non-linear second-order theory for the description of the position vector is introduced, which results in a normal acceleration correctly oriented along the normal vector of the deformed beam axis. For the sake of simplicity, the procedure is applied here only in the case of an initially straight beam.

In order to restrict the amount of variables and to concentrate on the benefits from the mixed formulation, the shear deformations due to shear-forces are neglected, and the system of partial differential equations in space and time is solved in Section 5 by means of a semi-analytical approach using the sinusoidal modal space of the curved beam. Thus, the beam is assumed to be a simply supported one with zero vertical displacements and torsional rotations at both boundaries.

The resulting time-variant ordinary differential equations are presented in Section 6 and then solved in Section 7 by means of a linear interpolation of all relevant quantities. The evaluation of Floquet's theorem is prepared in Section 8, while in Section 9, some typical results are presented and discussed. Finally, in Section 10, some conclusions are drawn.

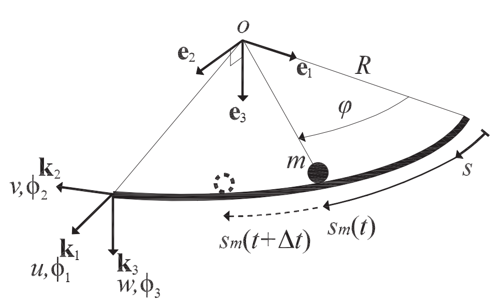

Fig. 1. Circle-like beam lying in a horizontal plane.

2. Formulation of the curved beam model

The curved beam/bridge is assumed to have a circle-like shape in the horizontal plane, spanned by the (fixed) Cartesian unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and perpendicular to the vertical unit vector $\mathbf{e}_3$.

2.1. Basic definitions

The coordinates and displacements of the beam points are defined locally by the following variables, clearly depicted in Figs. 1 and 2:

- R: radius, referred to the curved beam axis, i.e. to the cross-section centroid, G.
- r: radial position of a point, P, measured from the center of curvature O.
- φ: angular position of a point, measured from the center of curvature O.
- $s = R\varphi$: coordinate in circumferential direction along the curved beam axis.
- ξ: coordinate in the cross-section of the beam measured in radial direction from the centroid, G; the corresponding radial position r is: $r = R + \xi$.
- z: coordinate in the cross-section of the beam measured in vertical direction from the centroid.
- u: displacement in radial direction, defined by the local axis $\mathbf{k}_1$.
- v: displacement in tangential direction, defined by the local axis $\mathbf{k}_2$.
- w: displacement in vertical direction, defined by local/global vertical axis $\mathbf{k}_3$.
- ϕ_1: rotation around the local radial axis $\mathbf{k}_1$.
- ϕ_2: rotation around the local tangential axis $\mathbf{k}_2$.
- ϕ_3: rotation around the local/global vertical axis $\mathbf{k}_3 = \mathbf{e}_3$.

2.2. Kinematics and strains

Using the cylindrical coordinates r, φ, z, for any point of the beam the position vector $\mathbf{x}$ can be described with respect to the fixed initial Cartesian base $\mathbf{E} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$:

$$\mathbf{x} = r\cos\varphi\,\mathbf{e}_1 + r\sin\varphi\,\mathbf{e}_2 + z\,\mathbf{e}_3 \tag{1}$$

Adopting the comma notation for derivatives, (i.e. $\mathbf{x}_{,r} = \partial\mathbf{x}/\partial r$, etc.), the total differential, $\mathrm{d}\mathbf{x}$ of this position vector:

$$\mathrm{d}\mathbf{x} = \mathbf{x}_{,r}\,\mathrm{d}r + \mathbf{x}_{,\varphi}\,\mathrm{d}\varphi + \mathbf{x}_{,z}\,\mathrm{d}z \tag{2}$$

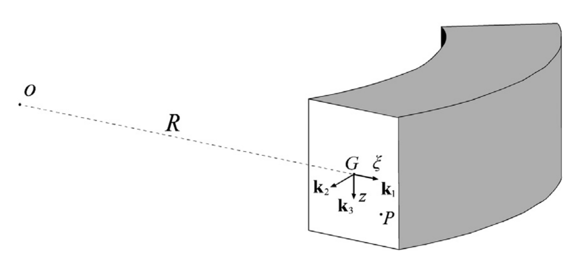

Fig. 2. Cross-section and local coordinates of the circle-like beam. O denotes the center of the beam axis, G the centroid and P a generic point of the cross-section.