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Experimental and numerical investigations of the responses of a cantilever beam possibly contacting a deformable and dissipative obstacle under harmonic excitation

Ugo Andreaus [a,*], Paolo Baragatti [a], Luca Placidi [b]

[a] Sapienza Università di Roma, Dipartimento di Ingegneria Strutturale e Geotecnica, Via Eudossiana 18, 00184 Roma, Italy
[b] International Telematic University Uninettuno, Engineering Faculty, Corso Vittorio Emanuele II 39, 00186 Roma, Italy

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ABSTRACT

In this paper, the dynamics of a cantilever beam subjected to harmonic excitations and to the contact of an obstacle is studied with the help of experimental and numerical investigations. The steel flexible structure is excited close to the free end with a shaker and may come into contact with a deformable and dissipative obstacle. A technique for modeling contact phenomena using piece-wise linear dynamics is applied. A finite-dimensional modal model is developed through a Galerkin projection. Concentrated masses, dampers and forces are considered in the equations of motion in such a way that the boundary conditions are those of a cantilever beam. Numerical studies are conducted by assuming finite-time contact duration to investigate the frequency response of the impacted beam for different driving frequencies. Experimental results have been extrapolated through a displacement laser sensor and a load cell. The comparison between numerical and experimental results show many qualitative and quantitative similarities.

The novelty of this paper can be synthetized in (a) the development of experimental results that are in good agreement with the numerical implementation of the introduced model; (b) the development of a comprehensive contact model of the beam with an unilateral, deformable and dissipative obstacle located close to the tip; (c) the possibility of accounting for higher modes for the cantilever beam problem, and hence of analyzing how the response varies when moving the excitation (and/or the obstacle) along the beam, and of investigating the effect of the linearly elastic deformability of the built-in end of the beam; (d) an easy and intuitive solution to the problem of accounting for spatially singular masses, dampers, springs and forces in the motion equations; (e) the possibility of accounting for finite gap and duration of the contact between beam and obstacle.

1. Introduction

Dynamic analysis of beam-like structures is significantly important in modeling real cases such as aircraft wings, spacecraft antennas, helicopter blades, robot arms and many other problems of practical interest. In this respect, almost all the structural mechanics courses and numerous studies can be found in the literature on the transverse vibrations of uniform beams under different types of standard boundary conditions [1]. However, in many real applications, the investigation of non-standard unilateral support conditions may provide a more realistic modeling that is desired for accurate structural analysis. Particularly, for structures with multiple and repeated impacts, the concentrated inertias, dampers, stiffnesses and forces affect the modal behavior of structures which cannot be neglected. Two examples are the workpiece–tool interactions during milling of thin-walled structures [2] and the tailoring of cutting forces, where periodic impacts are used for obtaining a desirable surface finish [3].

The problem of a cantilever beam impacting against an impact stop has also been considered by several authors, see for example references [4–6] and the references therein. An exact solution-set of the dynamics of a beam partially supported by an elastic foundation is also studied [7,8].

Common assumptions made in some of these studies include the following [9]:

- the response amplitude of the system is large compared to the amplitude of impactor motions;
- the impacted system's motion reverses after an impact;
- the impacted system is linear; and

* Corresponding author. Tel.: +39 6 44585297; fax: +39 6 44585292.
E-mail address: ugo.andreaus@uniroma1.it (U. Andreaus).

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- the time of contact between the impactor and the system is infinitesimal.

The dynamics of a vibro-impacting cantilever beam has been studied experimentally by several authors [10–15]. However, experimental results showed the importance of considering finite-time duration of the loading [9]. Dynamics of the cantilever beam has also been experimentally and analytically studied in some research works [1,16–18]. The frequency response of the systems has always been considered [19]. Non-linear dynamics of a cantilever beam has been studied considering the non-linearity due to the beam curvature and to the intermittent contact with another body [20–22]. In other works, linear dynamic models have been developed for contact AFM probes and numerically solved [23]. Impacts are always associated to concentrated forces and stresses. Therefore, the role of microstructures in terms of multi-phasic material [24,25] or on non-locality [26,27] should be taken into account. However, in the present study, the dynamics of a steel cantilever beam subjected to harmonic forcing against a deformable and dissipative obstacle is considered. For a range of values for the forcing frequency and amplitude, contacts between the beam and the obstacle may occur, resulting in vibro-contact motion of the beam. Attention is focused on the dynamics of the beam via experimentally recorded signals from the beam tip and from the obstacle. As shown in Fig. 1, the experimental apparatus is constituted by (i) an electro-magnetic shaker, to apply a harmonic excitation close to the cantilever tip, (ii) a noncontact laser sensor, which is used to measure the structure's displacement at the free end, and (iii) a piezoelectric load cell, to measure the force imparted to the obstacle by the beam at each contact. It is a matter of facts that contact forces have important applications in the design of machine parts or of structural components, which are subjected to contact loading. The first natural frequency of the structure is about 57 Hz, and the associated viscous damping factor is about 0.002. The range of excitation frequencies that is considered in the experiments is restricted to be less than 70 Hz; as a consequence, the excitation frequency is always below the second natural frequency of the structure. The aim is to predict and to interpret the dynamical behavior of the vibrating beam within this frequency region and to compare the results of experimental tests with those ones of numerical investigations, see e.g. [28].

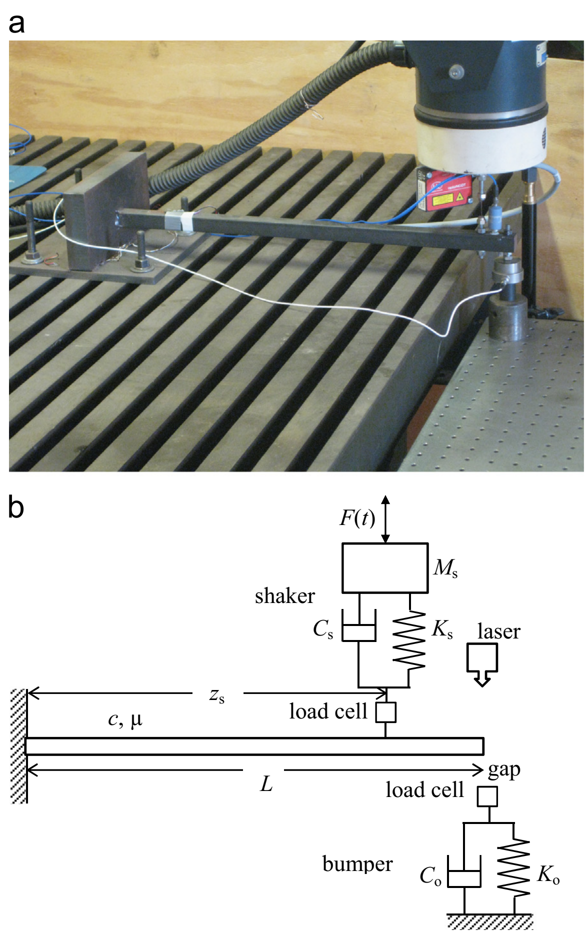

Fig. 1. Experimental set-up: (a) physical arrangement and (b) schematic idealization.

In this paper, the problem of modeling flexible beams subjected to contacts is addressed with the help of a multimode approach. In common with previous studies [5,29–33], a Galerkin approach is used to reduce the system of partial differential equations to a finite set of ordinary differential equations. However, in this work a technique is presented which allows to take into account on the one hand more than a single mode in the model, when multiple contacts occur between the beam and an obstacle located at a small finite distance from the beam itself, and on the other hand a finite-time duration of the contact. Moreover, the presence of concentrated masses, springs and dampers along the beam axis is contemplated as well as the application of contact external forces. In order to model the contact process, a piece-wise linear model of the beam based on the linearly viscous–elastic behavior of the bumper is formulated, making use of the Heaviside function. Finally, comparisons between experimentally recorded and numerically computed diagrams have been drawn. Qualitative and quantitative comparisons with experimental results using one or three degrees of freedom will be presented, and the issue of pseudo-resonance of the non-linear system is discussed with respect to different values of elastic stiffness and damping coefficient of the obstacle.

Two methods have been used in research works when including the force at the free end of the cantilever beam during dynamic analysis. One method is to consider the force at the end of the beam in the boundary conditions [9]. The other method is to consider the force of the obstacle to be a part of the equations of motion as an external datum and using some type of step functions such as the Heaviside and/or Dirac functions [34–40]. This research work investigates the beam vibration in tapping mode by means of the latter method and compares the results with experimental tests. The equations of motion are derived using Hamilton's Principle in the same way it is used in [41–44]. The modal shape functions, including the natural frequencies, are obtained using separation of variables; the time response is obtained using the Galerkin method. The obstacle is modeled by means of a spring and of a damper located near the free end of the cantilever; the contact between the beam and the obstacle is of the non-tension type, and therefore the obstacle delivers a piecewise linear force, which is obtained by the Heaviside function.

The innovative aspects which we introduce in this paper are represented by several respects, namely experimental displacement excursions and contact forces are in agreement with those numerically evaluated, the modeling of monolateral, linearly elastic and viscous obstacle; the inclusion in the model of higher modes of the cantilever beam in order to analyze how the response varies when moving the excitation (and/or the obstacle) along the beam, and to investigate the effect of the linearly elastic deformability of the built-in end of the beam; the possibility to consider point inertias, dampings and loads in the motion equations; finite gap between non-deformed beam and obstacle and finite duration of the contact are considered.