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Combination, principal parametric and internal resonances of an accelerating beam under two frequency parametric excitation

Bamadev Sahoo [a,*], L.N. Panda [b], G. Pohit [c]

[a] Department of Mechanical Engineering, International Institute of Information Technology, Bhubaneswar 751003, Odisha, India
[b] Department of Mechanical Engineering, College of Engineering and Technology, Bhubaneswar 751003, Odisha, India
[c] Department of Mechanical Engineering, Jadavpur University, Kolkata 700032, West Bengal, India

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ABSTRACT

This study analyses the nonlinear transverse vibration of an axially moving beam subject to two frequency excitation. Focus has been made on simultaneous resonant cases i.e. principal parametric resonance of first mode and combination parametric resonance of additive type involving first two modes in presence of internal resonance. By adopting the direct method of multiple scales, the governing nonlinear integro-partial differential equation for transverse motion is reduced to a set of nonlinear first order ordinary partial differential equations which are solved either by means of continuation algorithm or via direct time integration. Specifically, the frequency response plots and amplitude curves, their stability and bifurcation are obtained using continuation algorithm. Numerical results reveal the rich and interesting nonlinear phenomena that have not been presented in the existent literature on the nonlinear dynamics of axially moving systems.

1. Introduction

Dynamic behavior of an axially traveling beam have attracted much attention over many years as the response is affected by a number of parameters individually or their combinations. In engineering and industrial applications, traveling mechanical systems subjected to multi-frequency excitation is quite large. For example the belt spans used extensively in automobile engines experience multi-frequency excitations due to engine firing and accessory variable torques [1].

The literature on axially moving systems highlights various aspects of traveling media problems [2–9], namely type of models used (beam or string), the effects of different boundary conditions, parametric instabilities, various damping models for viscoelastic material and their effects on stability boundary and many more. Some recent publications relevant to the present investigations are presented here. Riedel and Tan [10] investigated the coupled longitudinal and transverse response of forced vibration of an axially moving strip. Sze et al. [11] studied the forced response of an axially moving beam with the fundamental, superharmonic and subharmonic resonant conditions. Marynowski [12] and Marynowski and Kapitaniak [13,14] studied the nonlinear dynamics of a traveling continuum considering various internal damping models for viscoelastic material. They compared between Kelvin model and Maxwell model [12]; Kelvin–Voigt and Bugers model [13] and concluded that for small values of internal damping, all models provide similar results. They [14] also studied the bifurcation and nonlinear dynamics of a traveling beam considering three parameter Zener internal damping model. Approximate solutions of a conveyor belt moving with a slow and time varying velocity was evaluated by Suweken and Van Horssen [15–17] considering a two time scales perturbation method.

Riedel and Tan [10] and Sze et al. [11] studied force responses of axially moving systems with internal resonance. Chen et al. [18] investigated forced nonlinear transverse vibration of an axially moving beam in presence of internal resonance employing multi-dimensional Lindstedt-Poincare (MDLP) method. They verified the results with that of Incremental Harmonic Balance Method (IHM) and concluded that the former method is more straightforward and efficient for a multi-degree of freedom system. Ozkaya et al. [19] analyzed non linear transverse vibration of an Euler-Bernoulli beam under 3:1 internal resonances considering multiple supports (3, 4, and 5). Bagdati et al. [20] extended the same work to find different resonance cases between different modes. Ozhan and Pakdemirli [21,22] applied the same method to develop a general solution procedure for the forced vibrations of a traveling system with cubic nonlinearities under primary resonances of external excitation with and without internal resonance. They performed the study for both elastic and viscoelastic beams. Huang et al. [23]

* Corresponding author.
E-mail addresses: bamadev@iiit-bh.ac.in (B. Sahoo), lnpanda@cet.edu.in (L.N. Panda), gpohit@mech.jdvu.ac.in (G. Pohit).

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and Ghayesh et al. [24] adopted IHM method and Galerkin method respectively to analyze nonlinear vibrations of an axially moving beam with constant speed and subjected to periodic lateral force excitations in presence of 3:1 internal resonance for the case of fundamental and subharmonic resonances [23] and primary resonance [24]. Chen and Tang [25] studied the nonlinear vibration of an axially accelerating viscoelastic beam subject to parametric excitation considering the effects of longitudinally varying tensions using method of multiple scales. Zhang et al. [26] employed Galerkin method to find equilibrium solutions of axially moving viscoelastic beam (Kelvin model) under simply supported end conditions in the super-critical speed regime subjected to harmonic forced excitation. Huang et al. [23] investigated bifurcation and chaotic behavior of an elastic beam traveling at constant speed using the IHM. They have considered geometric nonlinearity, but neglected longitudinal coupling effect. Recently Ghayesh, Amabili and co-researchers have made significant investigations on traveling continua problems considering various aspects. They employed continuation algorithm for steady state frequency response curves and direct time integration (Runge-Kutta) for dynamic solutions, viz. forced nonlinear dynamics of an axially moving beam under hinged-hinged condition with 3:1 internal resonance between first two transverse modes [24,27–29]. They modeled the traveling beam as integro-partial differential equation in [24] and coupled equations of motion in [27–29]. Ghayesh et al. [30] made similar studies [28] for a viscoelastic hinged-hinged beam of Kelvin–Voigt type and with no internal resonance. They applied modified Rosenbrock method for direct time integration. Ghayesh and Amabili [31,32] studied the nonlinear forced dynamics of traveling beam in the supercritical speed regimes with and without considering internal resonance for a buckled state of the traveling beam.

The above papers deal with beams traveling with either constant or variable axial speed and subject to single-frequency excitation. In most of the practical situations, systems are subjected to multi-frequency parametric excitations, for example the engine firing pulses in a multi-cylinder engine may cause belt translation speed to fluctuate with a periodic nature having two dominant Fourier harmonic components. Moreover, the excitations of an axially moving system are influenced by various reasons, such as, pulley eccentricities, irregular pulley radii, moments from the driving pulleys and driven accessories etc. In the present study, multi-frequency parametric excitation in the form of pulsating axial speed of the beam with typically two harmonic components and the consequent simultaneous parametric resonances in combination with 3:1 internal resonance between first two modes have been considered. It may be mentioned that the available literature on multi-frequency and parametric excitation on traveling systems is quite few. Nayfeh employed method of multiple scales for investigation on the two mode response of two-degree-of-freedom system subjected to two-frequency parametric excitations [33]; and the cubic nonlinear response of a single-degree-of-freedom system [34]. Further, Nayfeh and Jebril [35] made steady state analysis for a two-degree of freedom system with quadratic and cubic nonlinearity subject to multi-frequency parametric excitations. Plaut et al. [36] investigated a system of second order equations with weak quadratic and cubic nonlinearities. They used method of multiple scales to evaluate system response in the case of two frequency excitation and the influence of internal resonance. Parker and Lin [37] studied the dynamic stability of an axially moving media subjected to multi-frequency tension and speed fluctuations adopting a one term Galerkin discretization and perturbation method. El-Bassiouny [38] predicted the existence of steady state responses of a single degree of freedom nonlinear system with two-frequency parametric and self excitations using asymptotic techniques. Michon et al. [39,40] performed both analytical and experimental analysis of an axially moving belt subject to multi-frequency parametric excitation. Yang et al. [41] employed one term Galerkin discretization and the perturbation method to find approximate response of an axially moving viscoelastic beam subjected to multi-frequency external excitations. The numerical results were used to verify the approximate solutions. The authors of the paper [50,51] have made the stability, bifurcation and dynamic response study for a traveling beam under single-frequency parametric excitation in presence of 3:1 internal resonance using direct method of multiple scales.

From the existing literatures, it is clear that no investigations are available so far on traveling viscoelastic beams subject to two-frequency parametric excitations in conjunction with the internal resonance between first two modes. Hence, attempt has been made to address the lack of research in this area; the aim of this paper is to find the equilibrium solution and dynamic behavior of an axially moving viscoelastic beam subject to simultaneous principal parametric resonance of first mode and combination parametric resonance of additive type in presence of 3:1 internal resonance employing an analytic-numerical approach. Direct method of multiple scales is applied to the nondimensional nonlinear integro-partial differential equation of motion to get the complex modulation equations involving first two modes. These complex modulation equations are converted into a set of real valued first order partial differential equations. Such analytical process has been carried out following the steps outlined in Chin and Nayfeh [47]. The behavior of the system is determined by numerically solving these first order differential equations. The steady state solutions, their stability and bifurcations are determined using continuation algorithm [48]. Further, analysis on the effect of various system parameters on stability and bifurcation of equilibrium solutions of the system subjected to two-frequency excitation has been made in comparison with different single frequency parametric excitation cases. Moreover, direct time integration using variable step size Runge-Kutta method is applied to study the dynamic behavior in the form of stable periodic, mixed mode, quasiperiodic and unstable chaotic responses of the system.

2. Formulation of the problem

For the present work, a uniform horizontal beam traveling with a harmonically varying velocity is considered as shown in Fig. 1. The assumptions taken here are (1) the motion of the beam is planar, (2) the uniform cross sections of the beam remain plane during the motion and it behaves like an Euler–Bernoulli beam in transverse vibration, (3) the type of nonlinearity is geometric due to the midline stretching effect of the beam and (4) simply supported boundary conditions. The nonlinear integro-partial differential equation of transverse motion of the beam is given by (Wickert [45] and Chakraborty et al. [46]) and is reproduced below.

$$EI\frac{\partial^4 w^*}{\partial x^{*4}}+E^*I\frac{\partial^5 w^*}{\partial x^{*4}\partial t^*}+m\left(\frac{\partial^2 w^*}{\partial t^{*2}}+2v^*\frac{\partial^2 w^*}{\partial x^*\partial t^*}+\frac{\partial v^*}{\partial t^*}\frac{\partial w^*}{\partial x^*}+v^{*2}\frac{\partial^2 w^*}{\partial x^{*2}}\right)$$
$$-p\frac{\partial^2 w^*}{\partial x^{*2}}+c\frac{\partial w^*}{\partial t^*}=\frac{EA}{2L}\frac{\partial^2 w^*}{\partial x^{*2}}\int_0^L\left(\frac{\partial w^*}{\partial x^*}\right)^2 dx^* \quad (1)$$

with the associated boundary conditions

$$w^*(0,t^*)=w^*(L,t^*)=\frac{\partial^2 w^*}{\partial x^{*2}}(0,t^*)=\frac{\partial^2 w^*}{\partial x^{*2}}(L,t^*)=0 \quad (2)$$

The equation includes the nonlinearity due to midline stretching along with external viscous damping (Ghayesh et al. [27], Chakraborty and Mallick [42]), and viscoelastic internal damping (Ghayesh et al. [30], Yang et al. [41], Paidoussis [43]). The