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Analysis of steady-state response regimes of a helicopter ground resonance model including a non-linear energy sink attachment

B. Bergeot [a,b,*], S. Bellizzi [b], B. Cochelin [b]

[a] INSA Centre Val de Loire, Université François Rabelais de Tours, LMR EA 2640, Campus de Blois, 3 Rue de la Chocolaterie, CS 23410, 41034 Blois Cedex, France
[b] LMA, CNRS, UPR 7051, Centrale Marseille, Aix-Marseille Univ, F-13420 Marseille Cedex 20, France

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ABSTRACT

Helicopter Ground Resonance is a dynamic instability involving the coupling of the blades motion in the rotational plane (i.e. the lag motion) and the motion of the fuselage. This paper presents a study of the capacity of a Non-linear Energy Sink to control a Helicopter Ground Resonance. A model of helicopter with a minimum number of degrees of freedom that can reproduce Helicopter Ground Resonance instability is obtained using successively Coleman transformation and binormal transformation. A theoretical/numerical analysis of the steady-state responses of this model when a Non-linear Energy Sink is attached on the fuselage in an ungrounded configuration is performed. The analytic approach is based on *complexification-averaging* method together with *geometric singular perturbation theory*. Four steady-state responses are highlighted and explained analytically: *complete suppression*, *partial suppression through strongly modulated response*, *partial suppression through periodic response* and *no suppression* of the Helicopter Ground Resonance. A systematic method based on simple analytical criterions is proposed to predict the steady-state response regimes. The method is finally validated numerically.

1. Introduction

Ground Resonance (GR) is a potential destructive mechanical instability that can occur when a helicopter is on the ground and the rotor rotates. The phenomenon of GR involves a coupling between the fuselage motion on its landing gear and the blades motion in the rotational plane (i.e. the lag motion). It can be investigated without taking into account the aerodynamical effects. The standard reference of the GR analysis is the paper by Coleman and Feingold [1] where it is established that GR is due to a frequency coalescence between a lag mode and the fuselage mode. The range of rotors speeds Ω for which this frequency coalescence occurs is predicted analytically. More references can be found in [2–4] and a recent analysis of helicopter GR with asymmetric blades can be found in [5]. Traditionally, GR instability is prevented by two passive methods: increasing the damping [6] and modify the stiffness of the rotor blade lag mode or the fuselage mode. Active control of GR has been also studied in [4].

The Targeted Energy Transfer (TET) concept consists in controlling resonance by using an additional essentially non-linear attachment also named Non-linear Energy Sink (NES) to an existing primary linear system. TET has been extensively studied numerically, theoretically and experimentally, the results prove that the NES is very efficient for vibration mitigation [7] and noise reduction [8]. Impulsive loading was theoretically analyzed for example in [9] where TET is investigated in terms of resonance capture. In [10], harmonic forcing was considered where response regimes are characterized in terms of periodic and strongly modulated responses using an asymptotic analysis (multi-scale approach) of the averaged flow obtained using the complexification-averaging method [11]. In [12] a NES is used to reduce chatter vibration in turning process. An application of NES as a non-linear vibration absorber in rotor dynamics can be found in [13] where the efficiency of a collection of NES is analyzed for vibration mitigation of a rotating system under mass eccentricity force.

NESs are also used to control dynamic instabilities. The possible suppression of the limit cycle oscillations of a Van der Pol oscillator utilizing a NES is demonstrated numerically in [14]. In [15] (resp. [16]), the self-excitation response regimes of a Van der Pol (resp. Van der Pol-Duffing) oscillator with a NES are investigated. An asymptotic analysis of the system related to slow/super-slow decomposition of the averaged flow reveals periodic responses, global bifurcations of different types and basins of attraction of various self-excitation regimes. A series of papers [17–19] demonstrated that a NES coupled to a rigid wing in subsonic flow can partially or even completely suppress aeroelastic instability. In [17], the suppression mechanisms are investigated numerically. Several aspects of the suppression mechanisms are validated

* Corresponding author.
E-mail address: baptiste.bergeot@insa-cvl.fr (B. Bergeot).

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experimentally in [18]. Moreover, an asymptotic analysis is reported in [19] demonstrating the existence of the three passive suppression mechanisms based on TET. Suppression of aeroelastic instability of a general non-linear multi-degree of freedom system has also be considered in [20]. Finally, the discussion on relationship between dimensionality of the super-slow manifold, structure of the fixed points and the observed response regimes is explored in review paper [21].

In this context, the use of a NES appears to be an interesting alternative way to control GR instability which contrasts with the use of linear lag dampers having high damping value in order to suppress completely the dynamic instability. For its part, a NES attachment with a relatively small linear damping and a pure non-linear stiffness is able to prevent destructive amplitude of oscillations even if GR instability persists. This situations are hereafter referred as *partial suppression mechanisms*. The goal of the paper is therefore to study the effect of attaching an ungrounded NES on the fuselage of the helicopter for controlling GR instability. A number of the previous cited papers [14,17,7] use numerical methods to analyze the frequency interactions of this kind of essentially non-linear systems. They demonstrate that high-order resonances between the primary system and the NES may be very significant for adequate understanding transient dynamics in this class of non-linear systems. These details are beyond the scope of the present paper, since we focus on the characterization of the possible steady-state response regimes of a helicopter ground resonance model including a ungrounded NES attachment assuming a simple 1:1 resonance between the primary system (i.e. the helicopter model) and the NES.

The paper is organized as follows. In Section 2, the simplest helicopter model reproducing GR phenomenon is presented. It involves only lag motion of the four blades and one direction of the fuselage motion. Then, a NES is attached to the fuselage in an ungrounded configuration leading to the Simplest Helicopter Model including a NES (hereafter referred as SHM+NES). Preliminary results are presented in Section 3 including the linear stability analysis of the trivial solution of the SHM+NES. Moreover, using numerical simulations, the section presents also some steady-state response regimes which result from the NES attachment. We count four regimes classified into two categories depending on the fact that the trivial solution of the SHM+NES is stable or not. In Section 4, an analytical procedure based on complexification-averaging method together with geometric singular perturbation theory [22] is developed to analyze situations for which trivial solution of the SHM+NES is unstable. Finally Section 5 is dedicated to the prediction of the steady-state response regimes and numerical validation.

2. System under study

The system under study consists of a Simplest Helicopter Model (SHM) including a Non-Linear Energy Sink (NES). The SHM is first introduced.

2.1. Simplest Helicopter Model (SHM) that can describe ground resonance

To carry out the analytical approach presented in this work (in Section 4) we need to obtain a mechanical model of a helicopter which can reproduce the ground resonance phenomenon which has the minimum number of degrees of freedom (DOF). For that, a reference helicopter model, with 5 DOF (i.e 10 state variables in state-space) is first presented (Section 2.1.1). Next, it is simplified using successively Coleman transformation [1] (Section 2.1.2) and

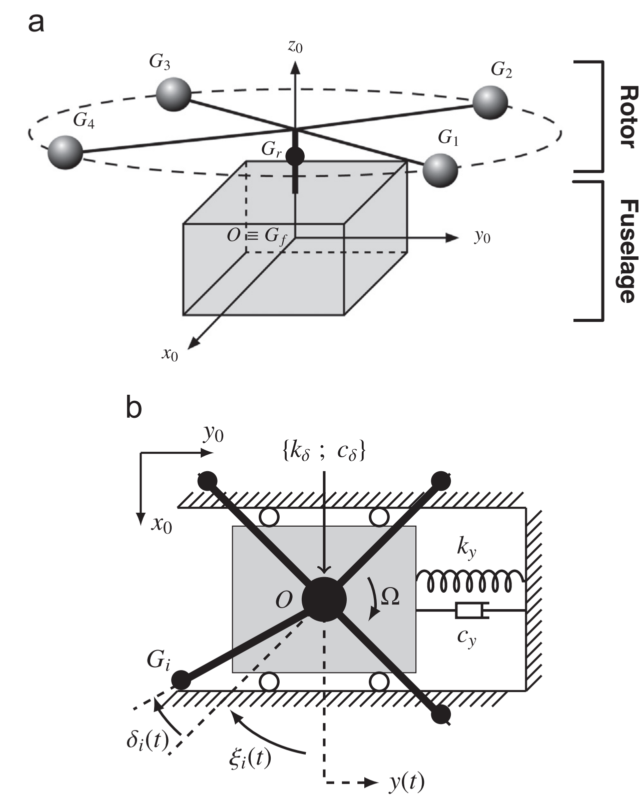

Fig. 1. Descriptive diagram of the used helicopter system. (a) Overview of the system. (b) View from the top.

binormal transformation [6] (Section 2.1.3) leading to the SHM which has 4 state variables in state-space.

2.1.1. Reference model

The reference model is very similar to that described for example in [2–4]. Here, it describes an idealized helicopter which consists of a fuselage on which a 4-blades rotor rotates at a constant speed Ω. Moreover, only lag motions are taken into account.

To obtain the equations of motion, a earth-fixed Cartesian coordinate system is considered where the origin, O, coincides with the center of inertia G_f of the fuselage at rest and the three Cartesian axes, x_0-axis, y_0-axis and z_0-axis, are shown in Fig. 1(a). At rest, the center of inertia of the rotor G_r is also located on the z_0-axis.

The fuselage is a simple damped mass–spring system with only one translational motion alone the y_0-axis characterized by the coordinate $y(t)$. Each blade is assumed to be a mass point G_i (with $i \in [1,4]$) placed at a distance L from the z_0-axis. The position of the ith blade in the x_0y_0-plane is therefore given by

$$\begin{cases} x_{G_i}(t) = L\ \cos\left(\xi_i(t)+\delta_i(t)\right) & \text{(a)} \\ y_{G_i}(t) = y(t)+L\ \sin\left(\xi_i(t)+\delta_i(t)\right), & \text{(b)} \end{cases} \quad (1)$$

where $\delta_i(t)$ is the lagging angle of the ith blade. The lagging angle is the angle between the current position of the blade and its equilibrium position $\xi_i(t) = \Omega t - (\pi/2)(i-1)$ (see Fig. 1(b)).

The equations of motion which govern the time evolution of the five degrees of freedom of the system (the fuselage displacement $y(t)$ and the four lagging angles $\delta_i(t)$) are then derived using