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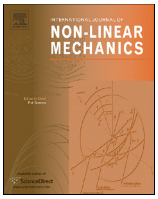

Two-dimensional modeling of unilateral contact-induced shaft precessional motions in bladed-disk/casing systems

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ABSTRACT

The present work targets shaft whirling motions induced by direct blade/casing unilateral contact occurrences in aircraft engine bladed-disk assemblies. These contact events are favored by increasingly reduced blade-tip clearances and potentially lead to harmful interactions that may threaten the engine structural integrity.

A simplified 2D in-plane finite element model representative of the engine fan stage is built, accounting for the flexibility of the shaft through two linear springs attached to the disk center node and the structural coupling provided by the fan frame and the bearings, modeled by an array of linear springs. A linear stability analysis of the reduced-order coupled system reveals two unstable zones in a selected rotational speed range, emanating from the linearly predicted modal coincidence speeds.

Through a time-marching strategy, two asymmetric contact initiation mechanisms are investigated: (1) a prescribed casing distortion and (2) a mass imbalance on the bladed-disk. It is shown how the 1-nodal diameter mode of the first modal family of the bladed-disk is dominant when a modal interaction arises from the transient casing distortion and leads to divergent regimes. The presence of the frame/bearings coupling induces a shift in the critical speeds detected, generally characterized by a backward traveling wave in the rotating frame and a forward traveling one in the fixed frame. Further, when a mass imbalance is the excitation source, the suspension modes appear to have a major role and a stable limit cycle is reached regardless of the coupling stiffness with much lower energy levels than in divergent regimes.

1. Introduction

In the aeronautical and space industry, reliability and efficiency stand as major concerns due to both economical and environmental reasons. Among other challenges, to gain fuel consumption while increasing power, lowering noise and gas emissions, increasing the durability of components while decreasing weight have attracted considerable interest. Thus, various strategies have been adopted in distinct fields, such as the use ultralight-weight materials (*e.g.* titanium alloys and composite materials) [1], planning cost-efficient flight trajectories [2] as well as the development of multidisciplinary optimization algorithms [3], to name a few. In the specific area of commercial aircraft propulsion, where thrust is often provided by a *gas turbine* as the one illustrated in Fig. 1, efficiency enhancement [4] is achieved through flow-path and geometry optimization, or compression ratios and combustion temperatures increase.

One of the most sensitive parameters to tune in compressor and fan stages is the *blade-tip clearance*, defined as the effective distance between the rotating airfoil tips and the surrounding stationary casing at nominal operating conditions. In fact, this distance must be kept to a minimum to ensure high compression ratios and engine performance [5]. As detailed in [6], potential structural malfunctions may arise from, for instance, mass imbalance or blade loss, impact with external objects, casing distortion due to high thermal gradients, fluid excitations through bearings or aerodynamic loads, among others. Due to the increasingly tight operating clearances, these incidents generally lead to involved shaft/bladed-disk/casing interactions induced by unilateral and frictional contact occurrences between rotating and stationary components. Such events that may threaten the engine structural integrity are commonly referred to as *structural rotor/stator interactions* [7,8].

Within this industrial framework, two types of systems must be distinguished depending on the contact interface involved in the interaction:

Rotor/bearing systems: The rotor is modeled by a flexible shaft and rigid disks, and supported by journal-bearings (Fig. 2a), *e.g.* magnetic [11] or hydrodynamic [12] bearings, liquid or gas annular seals [13], among others.

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Nomenclature

$(\bullet)^{bd}$	elements of $(\bullet)$ associated to the bladed-disk
$(\bullet)^{c}$	elements of $(\bullet)$ associated to the casing
$(\bullet)_{c}$	cosine component of double harmonics
$(\bullet)_{s}$	sine component of double harmonics
$\tilde{(\bullet)}$	elements of $(\bullet)$ in reduced-order space
$\boldsymbol{\Lambda}$	eigenvalues
$\boldsymbol{\Psi}_c$	constraint modes
$\boldsymbol{\Psi}_s$	static modes
$\mathbf{B}$	contact constraints matrix
$\mathbf{D}$	damping matrix
$\mathbf{K}$	stiffness matrix
$\mathbf{K}^*$	frame/bearings coupling matrix
$\mathbf{M}$	mass matrix
$\mathbf{V}$	eigenvectors
$\ddot{\mathbf{x}}$	accelerations
$\dot{\mathbf{x}}$	speeds
$\hat{\mathbf{u}}^k$	cyclic coordinates of k-th nodal diameter
$\mathbf{f}_{cn}$	contact forces
$\mathbf{f}_{ext}$	external forces

$\mathbf{q}$	modal coordinates in time-invariant space
$\mathbf{q}_{cb}$	Craig–Bampton modal participations
$\mathbf{u}$	modal coordinates
$\mathbf{x}$	displacements
$\mathbf{x}_b$	boundary DoF
$\mathbf{x}_i$	internal DoF
$\mathbf{z}$	state-space coordinates
Δt	time-step size
δ	initial blade-tip clearance
η	number of constraint modes
κ	coupling stiffness
Ω	rotational speed
Ω_k^{cr}	critical speed of kth nodal diameter
ω_k	eigenfrequency of kth nodal diameter
K	strain energy
k	nodal diameter
N	number of sectors
T	kinetic energy
t	time
x_o, y_o	center node DoF in rotating frame

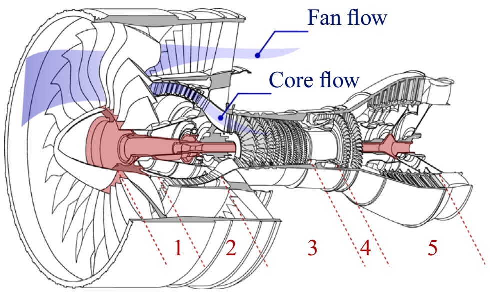

Fig. 1. Turbofan engine components: 1. fan; 2. Low Pressure (LP) compressor; 3. High Pressure (HP) compressor; 4. combustion chamber; and 5. HP and LP turbines.

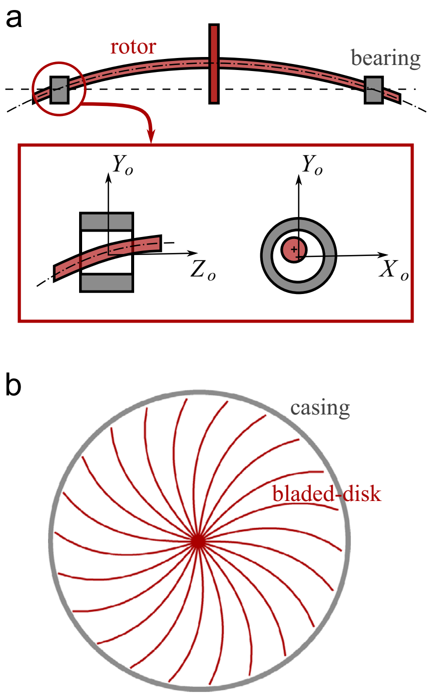

Fig. 2. Systems subject to structural rotor/stator interactions. (a) Rotor/bearing [9] and (b) Bladed-disk/casing [10].

Contacts shall occur between shaft and the inner part of its supporting bearings due to large lateral vibrations.

Bladed-disk/casing systems : A rigid shaft and flexible blades are commonly considered. Contacts take place between blade-tips and surrounding casings (Fig. 2b). The blades distortion and the casing wear are the main concerns [8,14].

Generally speaking, three distinct phenomena arising from rotor/stator interactions are discussed in the literature: (1) *modal coincidence* [15], characterized by an exchange of energy between modes of the rotor and the stator of matching nodal diameter through intermittent contacts that may lead to catastrophic structural failures [16], (2) *rubbing*, refers to direct contact either between shaft and journal bearings [7] or blade-tips and surrounding casing [17] depending on the system of interest and (3) *whirl* [9], describes precessional orbits of the shaft leading to unstable vibratory motions and is related to rubbing problems once the whirling amplitude is larger than the initial rotor/bearing clearance [7]. The rotor may undergo *forward* or *backward* whirl depending on the whirling direction with respect to the shaft rotational speed.

A majority of the numerical investigations on whirl involve flexible shafts with non-deformable cross-sections and the literature is rather scattered when considering whirling motions in fully flexible bladed-disks/casing systems. One of the only available studies [18] targets the transient response of a bladed Jeffcott rotor system. Three response configurations are reported: (1) no contact, (2) single blade contact and (3) multiple blades contact. Backward whirling motions are observed, maximizing blade displacements and stresses. A recent extension of this model was developed in [19], proposing a novel explicit formulation of the normal rubbing forces dependent on the system parameters such as the rotational speed, the blade cross-section and disk diameter, which was validated with experimental data from a test rig.

A fully flexible rotor is also developed in [20] using an energetic approach in the rotating frame and considering frictionless sliding contacts via a linear penalty method. The casing is modeled as an elastic ring and the stability of the two coupled structures is addressed through eigenvalue computations, showing different divergent instabilities and mode couplings. Parent et al. [21] built on this work, using a mass imbalance at the LP turbine as a source