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# On the flow of fluids through inhomogeneous porous media due to high pressure gradients

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ABSTRACT

Most porous solids are inhomogeneous and anisotropic, and the flows of fluids taking place through such porous solids may show features very different from that of flow through a porous medium with constant porosity and permeability. In this short paper we allow for the possibility that the medium is inhomogeneous and that the viscosity and drag are dependent on the pressure (there is considerable experimental evidence to support the fact that the viscosity of a fluid depends on the pressure). We then investigate the flow through a rectangular slab for two different permeability distributions, considering both the generalized Darcy and Brinkman models. We observe that the solutions using the Darcy and Brinkman models could be drastically different or practically identical, depending on the inhomogeneity, that is, the permeability and hence the Darcy number.



## 1. Introduction

The study of pressure driven flow through a porous slab is motivated by its relevance to the processes of enhanced oil recovery and carbon-dioxide sequestration. The flux due to the driving pressure is the quantity of greatest interest and the classical Darcy equation [1] is the most popular model used to determine it. One relevant aspect of modelling overlooked by the Darcy equation is the variation of material properties like density and viscosity with pressure. Experimental evidence of the variation of viscosity with pressure for various fluids can be found in [2–10]. In [11], a class of generalized models appropriate for studying flow through porous media under large pressure gradients was derived from thermodynamic considerations. Using a model that takes into account the pressure dependence of viscosity and drag, the shortcomings of a conventional approach (that assumes constant viscosity and drag) were highlighted by the results obtained in [12] and [13]. It was found that the flux does not increase indefinitely with driving pressure, but attains a ceiling value. A similar study of flow due to a pulsatile pressure gradient was also carried out in [14], again assuming a pressure dependent viscosity and drag coefficient. In these previous studies, the porous solid was assumed to be saturated and have uniform porosity, the flow was assumed steady, and inertial effects were neglected.

It is for the sake of simplicity that the porosity and permeability to flow are assumed uniform and equal in all direction, i.e., the porous medium is assumed homogeneous and isotropic with regard to its permeability. The permeability can then be represented by a constant scalar $k>0$. However, this is not true in general, for a porous medium may be heterogeneous. For example a bed of silt could be interspersed with inclusions of clay, sand or solid rock in which case the permeability to flow could change not only with spatial location but direction as well. The most general representation of the permeability is then in the form of a positive definite second order tensor that reduces to a diagonal form when the eigen directions are used as the basis. However, in the subsequent discussion, we shall focus on the case where the permeability is given by a scalar function $k(\boldsymbol{x})$ that depends on spatial location only, i.e., the permeability is the same in all directions.

The variation in permeability affects the flow of a fluid through the porous solid, and hence in problems of groundwater hydrology and petroleum engineering, the determination of the permeability distribution in a region is of great interest. The permeability distribution is estimated through experiments on samples, as well as from geo-statistical data.

Apart from its impact on the flow, the variation in permeability also poses a significant challenge to computational methods that seek approximate solutions to the equations that govern the flow of fluids through porous media. The variation of permeability usually occurs at a scale much smaller than the physical domain of the problem. The ratio of scales is so small that it is prohibitive, and sometimes impossible, to computationally obtain the field quantities with the same resolution as that of the given permeability

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data. In other words, the number of degrees of freedom required to discretize the domain at the scale of variation of the permeability is so large that solving the discretized system exceeds our computing power. This hurdle has spawned an entire field of research into the so called "multi-scale" or "reduced-order" approximations (see [15–17] for details).

However, our concern here is restricted to the features of the fluid response arising due to the inhomogeneity of the medium. In particular, we would like to get a qualitative idea of its effect on the flux in a pressure driven flow through a heterogeneous rectangular slab.

The slab, which is assumed rigid, has a permeability distribution $k(\boldsymbol{x})>0$. We shall consider a benchmark example of a permeability dataset that is used by the practicing petroleum engineers to simulate field conditions for the problem after we consider a simpler, idealized case of a rectangular region with a single circular inclusion in order to gain some clear insight into the flow of fluids through inhomogeneous porous media. The permeability is piecewise constant on the indicated regions $\Omega_0$ and $\Omega_1$. That is,

$$k(\boldsymbol{x})=\begin{cases} k_0 & \forall \boldsymbol{x}\in\Omega_0 \\ k_1 & \forall \boldsymbol{x}\in\Omega_1 \end{cases} \tag{1.1}$$

The interface to the two sub-domains is denoted by $\Gamma$. The left end is at a high pressure while the right end is at low (atmospheric) pressure.

Some problems of this type have been studied earlier. The steady flow of a Navier–Stokes fluid past a porous interface has been studied in [18,19]. The flow of fluid through and past a porous cylinder, and its variants, has been considered in [20,21]. These studies approached the problem from the vantage point of external flow, and the authors were interested in the question of drag due to the inclusion, the formation of a wake, separation of flow, etc. In these studies, two different flow regimes were considered, namely, the flow of clear fluid and flow through a porous medium. Hence, the authors had to deal with the question of boundary conditions at the interface. With a similar viewpoint, the flow through a porous medium with a rigid inclusion has been considered in [22]. However, because the inclusion is assumed impermeable, there is no flow through it, and the velocity is assumed to satisfy the no-slip condition on the surface. The work presented in [23] is the one most relevant in aims and scope to ours here, for the flow through a porous medium is considered, containing an elliptical inclusion with a different permeability. An exact solution is found using stream-functions and complex potentials and the flux is obtained as a function of the contrast (permeability ratio) and aspect ratio of the ellipse. However, the authors worked with a classical model with constant coefficients, which is what allowed them to find a closed form exact solution for the problem.

In contrast, we work with the generalized models that were used in [13,14], in which the viscosity and the drag coefficients are functions of pressure. We will be interested in the flux as before, and also the streamlines of the flow through the medium.

Our second example (Fig. 2) is drawn from the top layer of the benchmark dataset [24], and it substantiates the point made earlier that the permeability distribution in porous media can be very complex and show large contrast.[1]

[1] Contrast is defined as the ratio of the maximum and minimum permeability values.

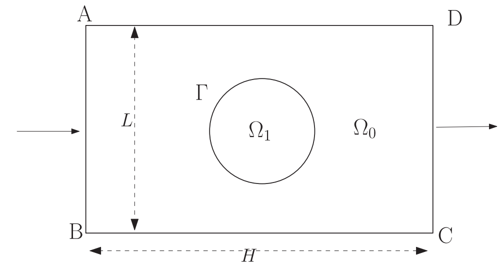


**Fig. 1.** The inhomogeneous slab *ABCD* is rigid and porous, hence permeability $k(\boldsymbol{x})=k_i\,\forall\boldsymbol{x}\in\Omega_i$ is discontinuous. High pressure at *AB* and low pressure at *CD* leads to a pressure gradient that drives the flow. No flow is possible through *BC* and *AD*.

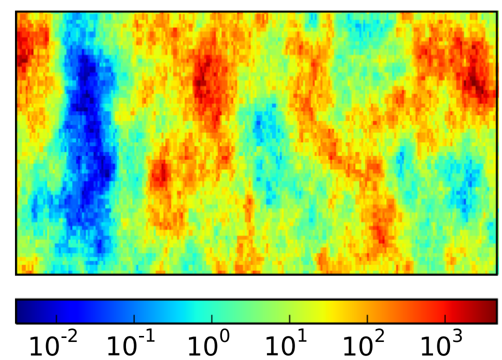


**Fig. 2.** Colourmap of the top layer of the inhomogeneous permeability dataset obtained from [24]. The permeability $k(\boldsymbol{x})$ is piecewise constant. Note the large contrast of the medium.

## 2. Governing equations

We neglect the effect of gravity on the flow, and assume that the fluid is incompressible. We also assume that the flow is steady and inertial non-linearities are negligible. The classical Darcy and Brinkman models are derived under a host of assumptions, and these models are not appropriate in applications involving large pressure gradients (see [13] for a detailed discussion of the relevant issues). Under high pressures, the viscosity of the fluid that appears in the model is not a constant, rather it varies exponentially with pressure.

This is represented by the Barus formula [25]:

$$\mu(p)=\mu_0\exp(\beta p), \tag{2.1}$$

where the exponent $\beta\geq 0$ depends on the fluid under consideration. From the experiments reported in [26–28] the measured value of the reciprocal of the pressure–viscosity exponent $\beta$ is seen to lie in the range 30–100 MPa for the fluids measured.

Hence we model the flow using particular instances of a generalized Darcy or generalized Brinkman model.

We denote by $\Gamma$ a curve in the domain across which the function $k(\boldsymbol{x})$ suffers a discontinuity.

### 2.1. The generalized Darcy equations

The generalized Darcy equation relates the pressure gradient to the velocity by

$$-\mathrm{grad}[p]=\frac{\mu(p)}{k}\boldsymbol{v}\quad \text{in } \Omega_0\cup\Omega_1 \tag{2.2}$$

$$\mathrm{div}[\boldsymbol{v}]=0\quad \text{in } \Omega_0\cup\Omega_1 \tag{2.3}$$

with boundary conditions

$$p|_{AB}=p_{\text{high}} \tag{2.4}$$