International Journal of Non-Linear Mechanics 63 (2014) 10–18

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Dynamical behavior of a mechanical system including Saint-Venant component coupled to a non-linear energy sink

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ARTICLE INFO

Article history:
Received 19 March 2013
Received in revised form
26 February 2014
Accepted 5 March 2014
Available online 14 March 2014

Keywords:
Energy exchange
Saint-Venant
Cubic
NES
Non-linear

ABSTRACT

Multi-scale vibratory energy exchange between a main oscillator including Saint-Venant term and a cubic non-linear energy sink is studied. Analytically obtained invariant manifold of the system at a fast time scale and detected fixed points and/or fold singularities at a first slow time scale let us predict and explain different regimes that the system may face during the energy exchange process. The paper will be accompanied by some numerical results confirming our analytical predictions.

1. Introduction

It has been proved that by endowing non-linear innate of some special light attachments, namely non-linear energy sink (NES), it is possible to localize the vibratory energy of important oscillators (which are mainly linear) or to suppress the chaos [1–6]. This localization can be for the aim of passive control and/or energy harvesting. The phenomenon is called energy pumping. The efficiency of NES systems in controlling main systems (e.g. in the field of acoustics, civil and mechanical engineering) has been proved experimentally as well [7–13]. Some works have been carried out in order to consider other types of non-linearities for the NES: Nucera et al. [14], Lee et al. [15] and Gendelman [16] studied energy pumping in systems with vibro-impact NES. The energy pumping in a two dof system consisting of a linear dof and a NES with non-polynomial potential investigated by Gendelman [17]. Lamarque et al. [18] pinpointed the energy pumping phenomenon from a linear master dof system to a non-smooth NES under different forcing conditions while the same system in the presence of the gravity was studied by Ture Savadkoohi et al. [19]. Some researchers took into account the vibratory energy exchange between a non-smooth main oscillator and a coupled non-smooth/cubic NES: Schmidt and Lamarque [20] by endowing techniques of [21–23] studied energy transfer from an initial single dof system including non-smooth terms of friction to a cubic NES. The behavior of two coupled non-smooth systems by detecting their invariant manifolds at different scales of time and finally their fixed point is enlightened in [24]. Ture Savadkoohi and Lamarque [25] analyzed dynamics and energy exchanges between a non-linear main structure of Dahl type and a non-smooth NES by detecting all possible fixed points and fold singularities and explaining different regimes of the system. In this paper we would like to analytically investigate on the multi-scale energy exchange between two oscillators, namely a main one including Saint-Venant term and a coupled cubic NES. Our analytical developments will be accompanied by some numerical examples. Organization of the paper is as follows: Academic model of the system, its re-scaling and averaging processes are given in Section 2. Time multi-scale behaviors of the system by detecting its invariant manifold and fixed points/fold singularities are described in Section 3. Some numerical results and their comparisons with our analytical developments are illustrated in Section 4. Finally, conclusions are given in Section 5.

2. The general presentation of the system

We consider a two dof system that is represented in Fig. 1. The main oscillator $(M, k, \tilde{\lambda})$ with a Saint-Venant type behavior $(\tilde{k}_p, \alpha)$ is coupled to a NES $(m, \tilde{C}_1, \tilde{\lambda}_1)$ with cubic potential. The mass ratio between two oscillators is very small, i.e. $0 < \epsilon = m/M \ll 1$.

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http://dx.doi.org/10.1016/j.ijnonlinmec.2014.03.002
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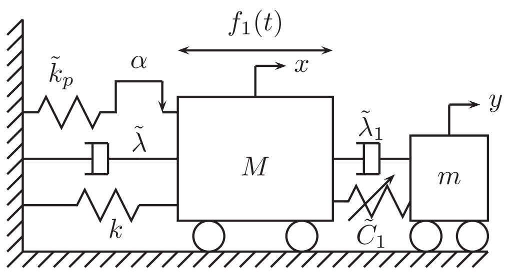

Fig. 1. Academic model of the system.

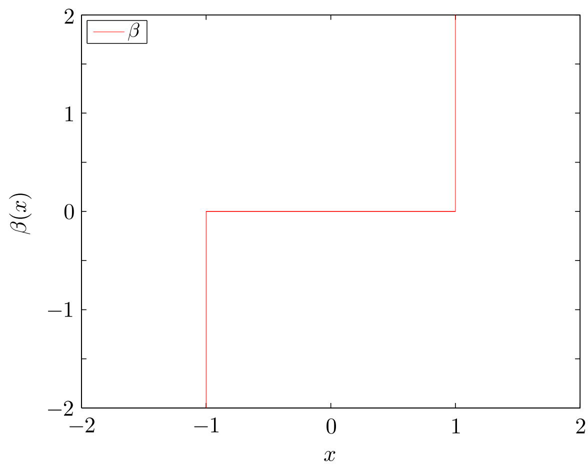

Fig. 2. Graph of β function in Eq. (1).

Governing equations of the described system in Fig. 1 are summarized as follows:

$$\begin{cases} M\dfrac{d^2x}{dt^2}+\tilde{\lambda}\dfrac{dx}{dt}+\tilde{\lambda}_1\left(\dfrac{dx}{dt}-\dfrac{dy}{dt}\right)+kx+\tilde{k}_pu+\tilde{C}_1F(x-y)=f_1(t) \\ m\dfrac{d^2y}{dt^2}+\tilde{\lambda}_1\left(\dfrac{dy}{dt}-\dfrac{dx}{dt}\right)+\tilde{C}_1F(y-x)=0 \\ \left(\dfrac{du}{dt}+\beta\left(\dfrac{\tilde{k}_pu}{\alpha}\right)\right)\ni\dfrac{dx}{dt} \end{cases} \tag{1}$$

with $F: s \mapsto s^3$ representing the cubic potential of the NES and $\ni$ refers a differential inclusion. The graph of β, which is illustrated in Fig. 2, is defined as follows:

$$\beta(x)=\begin{cases} \varnothing & \text{if } x\in]-\infty,-1[\cup]1,+\infty[\\ 0 & \text{if } x\in]-1,1[\\ \mathbb{R}_- & \text{if } x=-1 \\ \mathbb{R}_+ & \text{if } x=1 \end{cases} \tag{2}$$

Let us suppose that $\tau=t\sqrt{k/M}=w_1t$. We introduce the following variables: $\tilde{\lambda}w_1/Mw_1^2=\epsilon\lambda_0$, $\tilde{k}_p/Mw_1^2=\epsilon k_p$, $\tilde{C}_1/Mw_1^2=\epsilon C_{10}$, $\tilde{\lambda}_1w_1/Mw_1^2=\epsilon\lambda_{10}$, $f_1(\tau/w_1)/Mw_1^2=\epsilon f_{10}\ \sin(\Omega\tau)$, $\eta=\alpha/\tilde{k}_p$. System (1) reads (for any arbitrary function V, $dV/d\tau$ is denoted by V') as follows:

$$\begin{cases} x''+\epsilon\lambda_0x'+\epsilon\lambda_{10}(x'-y')+x+\epsilon k_pu+\epsilon C_{10}F(x-y)=\epsilon f_{10}\ \sin(\Omega\tau) \\ \epsilon y''+\epsilon\lambda_{10}(y'-x')+\epsilon C_{10}F(y-x)=0 \\ \left(u'+\beta\left(\dfrac{u}{\eta}\right)\right)\ni x' \end{cases} \tag{3}$$

Coordinates of the center of mass and relative displacement of two masses are introduced as follows:

$$\begin{cases} v=x+\epsilon y \\ w=x-y \end{cases} \Leftrightarrow \begin{cases} x=\dfrac{v+\epsilon w}{1+\epsilon} \\ y=\dfrac{v-w}{1+\epsilon} \end{cases} \tag{4}$$

System (3) in new coordinates is defined as follows:

$$\begin{cases} v''+\epsilon\lambda_0\dfrac{v'+\epsilon w'}{1+\epsilon}+\dfrac{v+\epsilon w}{1+\epsilon}+\epsilon k_pu=\epsilon f_{10}\ \sin(\Omega\tau) \\ w''+\epsilon\lambda_0\dfrac{v'+\epsilon w'}{1+\epsilon}+\dfrac{v+\epsilon w}{1+\epsilon}+\epsilon k_pu+(1+\epsilon)(\lambda_{10}w'+C_{10}F(w))=\epsilon f_{10}\ \sin(\Omega\tau) \\ \left(u'+\beta\left(\dfrac{u}{\eta}\right)\right)\ni\dfrac{v'+\epsilon w'}{1+\epsilon} \end{cases} \tag{5}$$

We assume that $\tilde{\tau}=\Omega\tau$ and then the following complex variables of Manevitch [26] are introduced to the system (5) (for any arbitrary function V, $dV/d\tilde{\tau}$ is denoted by $\dot{V}$)

$$\begin{cases} \phi_1e^{i\tilde{\tau}}=\Omega(\dot{v}+iv) \\ \phi_2e^{i\tilde{\tau}}=\Omega(\dot{w}+iw) \\ \phi_3e^{i\tilde{\tau}}=\Omega(\dot{u}+iu) \end{cases} \tag{6}$$

We endow the Galerkin method using a truncated Fourier series. In this paper we consider only the first harmonic, i.e. $e^{i\tilde{\tau}}$, for each equation. Based on the general periodic behavior of system variables, we obtain (see for example [19])

$$\begin{cases} \Omega\dot{\phi}_1-\dfrac{\Omega}{2i}\phi_1+\dfrac{\epsilon\lambda_0(\phi_1+\epsilon\phi_2)}{2(1+\epsilon)}+\dfrac{\phi_1+\epsilon\phi_2}{2i\Omega(1+\epsilon)}+\dfrac{\epsilon k_p}{2\Omega i}\phi_3=\epsilon\dfrac{f_{10}}{2i} \\ \Omega\dot{\phi}_2-\dfrac{\Omega}{2i}\phi_2+\dfrac{\epsilon\lambda_0(\phi_1+\epsilon\phi_2)}{2(1+\epsilon)}+\dfrac{\phi_1+\epsilon\phi_2}{2i\Omega(1+\epsilon)}+\dfrac{\epsilon k_p}{2\Omega i}\phi_3 \\ \quad+(1+\epsilon)\left(C_{10}\mathcal{F}+\dfrac{\lambda_{10}}{2}\phi_2\right)=\epsilon\dfrac{f_{10}}{2i} \\ \phi_3=\dfrac{\phi_1+\epsilon\phi_2}{(1+\epsilon)\pi}\xi\left(\dfrac{|\phi_1+\epsilon\phi_2|}{(1+\epsilon)\Omega}\right) \end{cases} \tag{7}$$

where

$$\forall z\in\mathbb{R}_+,\quad \xi(z) = \begin{cases} \pi & \text{if } z\le\eta \\ \theta+e^{-i\theta}\ \sin(\theta)-4e^{-i\theta/2}\ \sin\left(\dfrac{\theta}{2}\right)-\dfrac{4\eta}{z}e^{-i(\theta+\pi/2)} & \text{if } z>\eta \end{cases} \tag{8}$$

and

$$\theta=\arccos\left(1-\frac{2\eta}{z}\right) \tag{9}$$

We will analyze system behavior around 1:1 resonance (i.e. $\Omega=1+\epsilon\sigma$) by using a multiple scales approach.

3. Multi-scale analysis of the system

An asymptotic approach [27] will be used by introducing fast time τ_0 and slow times τ_1, τ_2,...

$$\tilde{\tau}=\tilde{\tau}_0, \tilde{\tau}_1=\epsilon\tilde{\tau}_0, \tilde{\tau}_2=\epsilon^2\tilde{\tau}_0, \ldots \tag{10}$$

so

$$\frac{d}{d\tilde{\tau}}=\frac{\partial}{\partial\tilde{\tau}_0}+\epsilon\frac{\partial}{\partial\tilde{\tau}_1}+\epsilon^2\frac{\partial}{\partial\tilde{\tau}_2}+\cdots \tag{11}$$

We study the system at different orders of ϵ.

3.1. ϵ^0 order

At the ϵ^0 order, the following equations can be derived from the system (7):

$$\frac{\partial\phi_1}{\partial\tilde{\tau}_0}=0\Rightarrow\phi_1=\phi_1(\tilde{\tau}_1) \tag{12}$$

$$\frac{\partial\phi_2}{\partial\tilde{\tau}_0}-\frac{\phi_2}{2i}+\frac{\phi_1}{2i}+C_{10}\mathcal{F}+\frac{\lambda_{10}}{2}\phi_2=0 \tag{13}$$